

**Mortality Modeling and Prediction:**

**A Comparative Analysis of the Cairns Blake Dowd (CBD) and Renshaw-Haberman (RH) Models**

**A research project submitted in partial fulfillment of the requirements of the degree of Bachelor of Science in Actuarial Science at the Jomo Kenyatta University of Agriculture and Technology.   
2023**

# DECLARATION

This research is our original work and has not been presented for a degree award in any other university.

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# LIST OF ABBREVIATIONS

LC: Lee-Carter

CBD: Cairns-Blake-Dowd

RH: Renshaw Haberman model

SVD: Singular Value Decomposition

OLS: Ordinary Least Squares

RMSE: Root Mean Squared Error

MAE: Mean Absolute Error

NCHS: National Center for Health Statistics

Contents

[DECLARATION 1](#_Toc162225892)

[ACKNOWLEDGEMENT 2](#_Toc162225893)

[LIST OF ABBREVIATIONS 2](#_Toc162225894)

[List of figures 6](#_Toc162225895)

[List of Tables 7](#_Toc162225896)

[CHAPTER ONE 8](#_Toc162225897)

[INTRODUCTION 8](#_Toc162225898)

[1.1 Background of the study 8](#_Toc162225899)

[1.2 Statement of The Problem 9](#_Toc162225900)

[1.3 Objectives 10](#_Toc162225901)

[1.4 Significance of the study 10](#_Toc162225902)

[CHAPTER TWO 12](#_Toc162225903)

[LITERATURE REVIEW 12](#_Toc162225904)

[2.1 Introduction 12](#_Toc162225905)

[2.2 Theoretical review 12](#_Toc162225906)

[2.3 Research gap 21](#_Toc162225907)

[CHAPTER THREE 22](#_Toc162225908)

[METHODOLOGY 22](#_Toc162225909)

[3.1 Summary 22](#_Toc162225910)

[3.2 Modelling Framework/Preliminaries 22](#_Toc162225911)

[3.3 Objective 1: To estimate mortality rates using the CBD and Renshaw-Haberman models from their current age in the year 2022 up to age 80. 24](#_Toc162225912)

[3.4 Objective 2: To construct cohort life tables, using the Renshaw-Haberman and CBD model. 29](#_Toc162225913)

[3.5 Objective 3: To compare the mortality trends based on the two models 30](#_Toc162225914)

[CHAPTER FOUR 33](#_Toc162225915)

[DATA ANALYSIS 33](#_Toc162225916)

[4.1 Data 33](#_Toc162225917)

[4. 2 Exploratory Data Analysis 33](#_Toc162225918)

[4.3 Objective 1: To estimate mortality rates using the CBD and Renshaw-Haberman models. 35](#_Toc162225919)

[4.4 Objective 2: To construct cohort life tables, using the mortality rates from the preferred model 49](#_Toc162225920)

[4.5 Objective 3: To compare the mortality trends based on the two models 67](#_Toc162225921)

[4.5.3 Longevity Risk 73](#_Toc162225922)

[CHAPTER 5 75](#_Toc162225923)

[Summary, Conclusions and Recommendations 75](#_Toc162225924)

[5.1 Summary 75](#_Toc162225925)

[5.2 Conclusion 76](#_Toc162225926)

[5.3 Recommendations 77](#_Toc162225927)

[5.4 Limitations 77](#_Toc162225928)

[REFERENCES 79](#_Toc162225929)

[R Code Excerpt 83](#_Toc162225930)

[WORK PLAN 89](#_Toc162225931)

[BUDGET 90](#_Toc162225932)

# List of figures

[Figure 1: Death Rates against age 33](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223788)

[Figure 2:Death Rates against age over different years 34](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223789)

[Figure 3:RH female Parameters 36](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223790)

[Figure 4: RH male parameters 38](#_Toc162223791)

[Figure 5: CBD female parameters 40](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223792)

[Figure 6: CBD male parameters 41](#_Toc162223793)

[Figure 7:RH model Residuals Females 45](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223794)

[Figure 8: CBD model Residuals Females 45](#_Toc162223795)

[Figure 9: RH model Residuals Males 46](#_Toc162223796)

[Figure 10: CBD Residuals Males 46](#_Toc162223797)

[Figure 11: Probability of dying within the next t years against age for 1981 cohort 67](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223798)

[Figure 12:Proportion of individuals that survive to age x against age for 1981 cohort 68](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223799)

[Figure 13: Life expectancy against age, for 1981 cohort 69](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223800)

[Figure 14: Probability of dying within the next t years, for 2002 cohort 70](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223801)

[Figure 15: Proportion of individuals that survive to age x for 2002 cohort 71](#_Toc162223802)

[Figure 16: Life expectancy against age for 2002 cohort 72](#_Toc162223803)

[Figure 17: Life expectancy against time, for Longevity Risk 73](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223804)

[Figure 18: Work plan 89](#_Toc162223805)

[Figure 19: Budget 90](#_Toc162223806)

# List of Tables

[Table 1:RMSE Estimates 42](#_Toc162223852)

[Table 2:MAE Estimates 43](#_Toc162223853)

[Table 3: Female mortality forecasts from RH model 48](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223855)

[Table 4:Male mortality forecasts from RH model 48](file:///C:\Users\Admin\Downloads\Modeling%20Mortality%20with%20Cairns%20Blake%20Dowd%20(CBD)%20and%20Renshaw-Haberman%20Models_%20A%20Comparative%20Analysis%20of%20the%20mortality%20trends%20%20%20(1).docx#_Toc162223854)

[Table 5: 1981 Female Life Table Generated using Mortality Rates of RH 50](#_Toc162223856)

[Table 6:1981 Male Life Table Generated using Mortality Rates of RH 52](#_Toc162223857)

[Table 7: 2002 Female Life Table Generated using Mortality Rates of RH 54](#_Toc162223858)

[Table 8: 2002 Male Life Table Generated using Mortality Rates of RH 56](#_Toc162223859)

[Table 9: 1981 Female Life Table Generated using Mortality Rates of CBD 58](#_Toc162223860)

[Table 10: 1981 Male Life Table Generated using Mortality Rates of CBD 60](#_Toc162223861)

[Table 11: 2002 Female Life Table Generated using Mortality Rates of CBD 62](#_Toc162223862)

[Table 12: 2002 Male Life Table Generated using Mortality Rates of CBD 64](#_Toc162223863)

[Table 13: Estimated life expectancy against expected life expectancy 66](#_Toc162223864)

# CHAPTER ONE

## INTRODUCTION

### 1.1 Background of the study

Mortality trends, refer to the patterns and changes observed in the central death rate, life expectancies, cumulative death rate, survivorship function, total number of years lived and probability of death within a population over a specific period of time. Studying these trends is useful in the evaluation of the accuracy of the different models of mortality prediction. Over the years, there have been various models that have been developed to try to ascertain the changes in mortality rates in the future. The most well-known of these models is the Cairns-Blake-Dowd (CBD) and Lee Carter (LC) model, which researchers have modified and adjusted to account for various effects on mortality. One adjustment of the Lee-Carter model is the modification of the original Lee-Carter model to account for cohort effects. Therefore, bringing about the Renshaw Haberman model. Similarly, the Cairns-Blake-Dowd model has undergone various modifications, one being the addition of the cohort effect to the original CBD model to bring about the CBD (2) model also known as the M6 model.

According to Withers (2009), a cohort is a set of individuals that are grouped according to similar traits which in most cases is by age, in other words, a birth cohort. Cohort analysis is following a specific group of people born in a specific year and studying them to track their behavioral change. The data from cohort analysis can be used to generate cohort life tables. The primary reason for studying cohort mortality is to have an understanding of how certain external and internal factors affect cohort mortality (Thelle & Laake, 2015). As a result of such studies, one can be able to get a preview of how similar internal and external factors can affect the mortality of the demographic group as a whole.

In the construction of life tables, the period and cohort approaches are used to analyze and understand mortality. Period life tables focus on examining demographic data within specific intervals or periods while cohort centers on analyzing data for groups of individuals born during the same time period and then following these cohorts throughout their lifetimes. Through life tables, there’s the ability to observe mortality patterns, obtain life expectancy for different cohorts, and obtain survival probabilities and population projections which are important for summarizing mortality data (Glenn, 1977).

The study seeks to model mortality for the United States of America population using the Renshaw Haberman and CBD models then perform a comparative analysis of the two models. The motivation for picking the Renshaw-Habberman and CBD models is that both the Renshaw-Haberman and CBD models are widely recognized with widespread adoption in academic research and practical applications make them relevant for comparative evaluation.

In this study, two particular cohorts of interest are considered; the 1981 cohort and 2002 cohort. The choice to analyze the 1981 and 2002 cohorts for mortality modeling and comparative analysis is particularly significant due to their status as the largest active population cohorts today. These cohorts represent substantial segments of the population actively engaged in workforce participation, economic activity, and social interactions. Given their size and demographic significance, understanding the mortality trends within these cohorts holds paramount importance for various stakeholders, including actuaries. Actuaries rely on accurate mortality projections to assess risks, determine insurance premiums, and effectively manage pension funds. By focusing on cohorts that constitute the largest active population today, this analysis provides invaluable insights into mortality dynamics that directly influence financial planning, healthcare strategies, and broader socio-economic policies.

### 1.2 Statement of The Problem

Accurate mortality rate predictions are pivotal for guiding decisions in life insurance. It is widely accepted in actuarial science that, for the pricing of and reserving for annuity and pension products, we need to understand the trends in mortality rates over time so that the underlying changes can be accurately modeled and projected into the future. These trends have been predominantly downward for the populations of many countries in recent years. A failure to account for these downward trends would mean that the premiums and reserves for annuity and pension products would be understated with potentially disastrous consequences for the financial institutions involved.

The existing divergence in approaches and assumptions among widely used models for mortality predictions, such as the Renshaw-Haberman and Cairns-Blake-Dowd (CBD) models, raises the debate as to which model gives predictions closer to actual mortality. This necessitates a thorough examination of their performance. By systematically evaluating and comparing these models, this research seeks to enhance the precision of mortality predictions and assess the performance of these two models in predicting mortality rates.

### 1.3 Objectives

**Main Objective**

To Model mortality with Cairns Blake Dowd (CBD) and Renshaw-Haberman Models: a comparative analysis of the mortality trends.

**Sub objectives**

1. To estimate mortality rates using the CBD and Renshaw-Haberman models.
2. To construct cohort life tables, using the mortality rates from the two models
3. To compare the mortality trends based on the two models.

### 1.4 Significance of the study

This study holds great significance within the fields of actuarial science and demographic research. The outcomes of this study will not only assist professionals in actuarial science, providing them with more reliable tools for risk assessment, but will also offer valuable insights to researchers, policymakers, and academics. In pension schemes**,** predicting mortality rates allows them to estimate how long retirees are likely to live and, consequently, how long they will be drawing pension benefits. This information is essential for determining the amount of money needed to fund the pension obligations.

In life insurance,mortality rates are used in longevity risk management which is the risk of policyholders living longer than expected resulting in increased payout amount. Accurate predictions will allow for better reserve calculation ensuring the insurer's financial stability. This study will contribute to the continuous improvement of mortality prediction methodologies, thereby shaping the landscape of actuarial practice and demographic analysis. This study also contributes to the body of knowledge and is a stepping stone for more research on the prediction of mortality.

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# CHAPTER TWO

## LITERATURE REVIEW

### 2.1 Introduction

This literature review explores the history and developments in mortality, mortality projection, and history of life tables. It examines various studies, methodologies, and models employed in the field of mortality projections. The findings contribute to a comprehensive understanding of the theoretical and empirical foundations of mortality prediction techniques and identifying research gaps from past studies.

### 2.2 Theoretical review

#### 2.2.1 Overall Mortality In US

The United States has enjoyed more than a century of nearly uninterrupted declines in mortality and rising longevity (Wharton PPI, 2016). In 1900, 1 in 40 Americans died annually. By 2013, that rate was roughly 1 in 140, a cumulative improvement of more than two thirds.Life expectancy at birth rose by more than 30 years over this period, from 47 to 79.While the overall pace of mortality decline has been fairly steady, its causes have varied over time. In the early part of the twentieth century, public health measures and improved nutrition led to rapid reductions in mortality caused by infectious diseases. As a result, cardiovascular disease and cancer accounted for nearly three quarters of all deaths by 1950. Beginning in the 1960s, medical advances and changes in people’s behavior turned the tide on these diseases, and mortality improvements since then have been driven primarily by better treatment and disease management (Wharton PPI, 2016).

Also, according to Wharton, in the twentieth century, the United States witnessed an unprecedented decrease in mortality rates. From 1900 to 2013, life expectancy at birth surged by over 30 years, while the overall death rate steadily declined by approximately 1% annually. However, beneath this overarching trend lies significant shifts in the root causes of mortality. At the dawn of the twentieth century, infectious diseases reigned as the primary contributors to mortality, responsible for nearly one-third of all deaths. The rapid urbanization and burgeoning population density of preceding decades provided fertile ground for bacterial dissemination, resulting in high mortality rates from diseases such as tuberculosis, influenza, and waterborne illnesses. Nonetheless, over the initial decades of the 1900s, concerted public health efforts, advancements in nutrition, and innovations in medical technology markedly curbed infectious disease-related fatalities. Initially, this decline was propelled by urban sanitation initiatives, including the implementation of water filtration and chlorination systems in major cities, alongside enhanced resistance to infections stemming from improved nutritional practices. With the advent of antibiotics in the mid-1930s, the pace of mortality rate improvement surged even further.

Despite the steady decline in overall mortality, differences persist between demographic and socioeconomic groups. Women live longer than men, while whites and Hispanics live longer than blacks. According to Latoya Hill& Samantha Artiga(2023)**,**despite substantial improvements in life expectancy among various racial and ethnic groups, persistent disparities have endured over time. Historically, Black individuals have consistently experienced shorter life expectancies compared to White counterparts, whereas Hispanic individuals have consistently enjoyed longer life expectancies than their White counterparts. For instance, at its peak in 2014, life expectancy for Black individuals lagged behind White individuals by more than three years (75.3 years compared to 78.8 years), while Hispanic individuals surpassed White individuals with a life expectancy of 82.1 years.

But the largest disparities in life expectancy reflect differences in educational attainment. On average, individuals with a college or advanced degree live more than 10 years longer than those without a high school degree. According to Hummer & Hernandez(2013), data from the United States revealed significant disparities in mortality rates among adults aged 25 to 64 based on educational achievement, particularly among white women and men. White women who had not finished high school experienced a mortality rate nearly four times higher than those with 16 or more years of education. Similarly, the gap widens among white men with the same educational backgrounds, as those without a high school diploma exhibited a mortality rate over four times greater than those with at least 16 years of education. Intermediate mortality rates were observed among individuals with a high school diploma or some college education. Thus, each increment in educational attainment correlated with a reduced mortality rate for both white women and men.Moreover, while gaps in life expectancy between men and women and between racial groups have narrowed in recent decades, differences by educational attainment have grown larger.

In 2016, children and adolescents (1 to 19 years of age) represented a quarter of the total estimated U.S. population; reflecting relatively good health, they accounted for less than 2% of all U.S. deaths.By 2016, death among children and adolescents had become a rare event. Declines in deaths from infectious disease or cancer, which had resulted from early diagnosis, vaccinations, antibiotics, and medical and surgical treatment, had given way to increases in deaths from injury related causes, including motor vehicle crashes, firearm injuries, and the emerging problem of opioid overdoses. Although injury deaths have traditionally been viewed as “accidents,” injury prevention science that evolved during the latter half of the 20th century increasingly shows that such deaths are preventable with evidence-based approaches(Cunningham et al., 2018).

According to Korhonen(2024) in the United States in 2020, the death rate was highest among those aged 85 and over, with about 16,354.9 men and 14,560 women per 100,000 of the population passing away. For all ages, the death rate was 1,090.8 per 100,000 of the population for males, and 965.1 per 100,000 of the population for women. According to the National Center for Health Statistics(Xu et al., 2022) ,life expectancy at birth for females in the USA was 76.4 years for the total U.S. population, a decrease of 0.6 from 77.0 years in 2020. For males, life expectancy decreased by 0.7 from 74.2 in 2020 to 73.5 in 2021.

In 2022, approximately 3,273,705 deaths occurred in the United States. The estimated 2022 age-adjusted death rate decreased by 5.3%, from 879.7 per 100,000 persons in 2021 to 832.8. COVID-19 was reported as the underlying cause or a contributing cause in an estimated 244,986 (7.5%) of those deaths (61.3 deaths per 100,000). The highest overall death rates by age, race and ethnicity, and sex occurred among persons who were aged ≥85 years, non-Hispanic American Indian or Alaska Native, non-Hispanic Black or African American (Black), and males. In 2022, the four leading causes of death were heart disease, cancer, unintentional injuries, and COVID-19(Ahmad, 2023).The 2022 rate of 5.6 infant deaths per 1,000 live births is the highest since 2018. (2020’s rate of 5.42 was the all-time low for the country.) Though one year of notable change in the opposite direction doesn’t necessarily indicate a new continuing trend, it’s enough to spark concern for those who study infant and maternal health (WEISS, 2023).

While the upward trend in life expectancy brings undeniable benefits to individuals, it also poses financial challenges for federal programs like Social Security and Medicare. Longer life spans lead to an increased number of individuals receiving federal retirement benefits, as well as extending the duration of time over which they receive these benefits. Consequently, forecasting mortality rates plays a crucial role in projecting federal budgetary outcomes. While most analysts anticipate mortality rates to continue decreasing at a pace similar to recent decades, the Social Security Trustees predict a notable slowdown in this trend. This more cautious outlook regarding life expectancy contributes to a considerably more positive assessment of Social Security's financial outlook.

**2.2.2 Causes of mortality**

Altogether, the death rate from infectious disease fell by 90 percent from 1900 to 1950, accounting for nearly two thirds of the overall reduction in mortality during that period.

As deaths from infectious diseases declined in importance, chronic diseases emerged as the leading killers. Driven in part by the steady rise in smoking rates, mortality from cardiovascular disease and cancer rose more than 50 percent from 1900 to 1950, together accounting for more than three quarters of all deaths by the middle of the century(Wharton PPI, 2016).

Also according to Wharton (2016),cardiovascular mortality peaked in 1950 and began to decline rapidly by the mid-1960s, falling roughly by half from 1960 to 1990. Medical advances during this period produced more effective treatments for cardiovascular disease and especially for acute heart disease, including specialized coronary care units, new pharmaceuticals, and advanced surgical procedures such as bypass surgery and angioplasty. A more active approach to prevention and disease management, as well behavioral changes such as reduced smoking rates and improved diet, further reduced the likelihood of severe heart disease and the associated mortality risk.

Aggregate mortality fell by one-third from 1950 to 1990, with nearly all of that decline attributable to cardiovascular diseases and more than half to heart disease alone.In 2022, the four leading causes of death were heart disease, cancer, unintentional injuries, and COVID-19(Ahmad, 2023).

According to Statista(2022),approximately 3.46 million people in the United States lost their lives in 2021. The most common causes of death in the U.S. were heart disease and cancer, however, COVID-19 accounted for around 12 percent of all deaths that year, making it the third leading cause of death. Due to the high number of deaths caused by COVID-19, the years 2020 and 2021 saw a substantial increase in overall deaths compared to previous years. The odds of someone dying from heart disease or cancer in the United States in 2021 were 1 in 6 and 1 in 7, respectively, while the odds of dying from COVID-19 were 1 in 10. Although heart disease and cancer currently account for the vast majority of deaths in the United States, this was not always true. In the early 1900s, pneumonia or influenza, tuberculosis, and gastrointestinal infections were the leading causes of death. Nowadays, deaths attributed to pneumonia or influenza remain common, but health issues such as diabetes, Alzheimer’s disease, and suicide are major causes of death.

While cancer deaths continued to rise at a gradual pace throughout this period, mortality from cancers peaked in 1990 and then began to fall steadily. Some of this decline is attributable to the reduced prevalence of smoking, and many credit this trend to successful public health campaigns and the imposition of tobacco control. However, medical innovation likely played a greater role, as declines in mortality from specific types of cancer were systematically related to the quality of pharmaceuticals, imaging technology, and surgical procedures used in the treatment of that particular cancer. By one estimate, new drug treatments and medical technologies explain more than three quarters of the decline in cancer deaths since 2000.

#### 2.2.3 Mortality Prediction

Mortality prediction has been an area of much interest, evolving significantly over time. The roots of mortality modeling trace back to the early 18th century, characterized by subjective models that relied on expert opinions rather than data-driven approaches. De Moivre (1725) proposed constructing life tables from mortality datasets using linear survival functions, marking an early attempt to formalize mortality analysis. This was followed by Gompertz's (1825) demonstration that mortality follows an exponential pattern across all ages, highlighting the acceleration of death rates with an increase in age. This foundational insight laid the groundwork for subsequent studies in mortality modeling. The works of Brass (1971) and Wilmoth (1990) further refined models using logistic and logarithmic transforms to ensure positive mortality rates.

Lee and Carter's groundbreaking work (1992) marked a paradigm shift in mortality prediction, pioneering the introduction of the influential Lee-Carter model. This two-factor model became a cornerstone in demographic literature and sparked the development of various extensions and adaptations to address its limitations (Deaton and Paxson 2004).

Recognizing the limitations of the Lee-Carter model, researchers sought to refine its assumptions and broaden its applicability. Brouhns et al. (2002) and Giacometti et al. (2009) explored alternative distributions, like the Poisson and generalized hyperbolic distributions, to mitigate the normality assumption in the Lee Carter random component. Additionally, Mitchell et al. (2013) proposed the Mitchell-Brockett-Mendoza-Muthuraman (MBMM) model by applying parameterization to the detrended natural logarithm of mortality rates.

The earliest and generally famous stochastic factor discrete-time model, Lee and Carter (1992) postulate that the true underlying death rate =- log(1−). This implies that longevity risk is not affected by cohorts i.e. changes in age-specific demographic parameters, and lower infant mortality rates. The Renshaw and Haberman (2006) was an extension of the Lee-Carter model effects. This modification to the Lee-Carter model was done to capture the effects that could be attributed to the year of birth (t −x). This model has been generally used for both demographic and actuarial applications because, firstly, it produced satisfactory fits and forecasts of mortality rates for different nations. For instance, the Lee-Carter model was used in Japan, Austria, Australia, Belgium, and the Nordic countries. Secondly, the Lee-Carter model structure permits the construction of confidence intervals related to mortality projections. Despite its reasonable performance, the LC model had a few constraints (Lee 2000) which caused negative responses. Because of this, new stochastic models were produced with the most remarkable models being the Renshaw and Haberman (2006) and Cairns et.al models (2006, 2007, and 2008).

More significant advancements reviewed the parameterization of the Lee-Carter model. Renshaw and Haberman (2006) introduced the Age-Period-Cohort (APC) model, also known as the Renshaw Haberman model, by incorporating a cohort effect, whereas Cairns et al. (2006, 2007) elaborated on the implementation of a Generalized Linear Model (GLM) for mortality odd ratios, resulting in the well-known Cairns-Blake-Dowd (CBD) model. The CBD model is a two-factor model that posits each of its two parameters follows a random walk with drift. In this model, the rate of drift is constant, and the changes in the parameters are correlated. The CBD model describes the logit of the initial mortality rate with a slope term and an intercept term, allowing for the number of deaths to follow a Poisson distribution. Future stochastic simulations are then obtained by projecting these two terms as following correlated random walks.

A study by Cairns et al., (2009) sought to use formal methods of model selection to rank which of the eight mortality models works best for data on males in England-Wales between 1961 to 2004 and males in the United States from the period 1968 to 2003. The focus of this study was mainly on males of higher ages 60 to 89 since the study was interested in the mortality rates of pensioners whose mortality rates had been declining over the years. In order to achieve this, Cairns et al., (2009) used various methods of mortality selection such as the Bayes Information Criterion (BIC), robustness in parameters, standardized residuals, and the comparison of nested models. From this, Cairns, et. al, (2009) concluded that no specific model stands out as being better than others. However, they observed that different models have different strengths in the way they project mortality.

Some of the strengths include; the Renshaw-Haberman model allows for great flexibility of the age effects while the one-dimensional P-splines method allows for smoothing of the age effects if the effects are seen as a drawback to estimating mortality. Similarly, the CBD model and its extensions were also seen to allow the smoothing of age effects but in contrast to the Renshaw-Haberman model they allow for richer period effects. Based on all these strengths Cairns et al., (2009) concluded that according to the BIC, the best model for the England-Wales data would be an extension of the CBD model that assumes that the impact of the cohort effect for any specific cohort diminishes over time instead of remaining constant. Moreover, for the United States dataset, the preferred model would be The Renshaw and Haberman model. However, based on the robustness of parameter estimates, a generalized model of the CBD that adds a quadratic term to the age effect was seen as the perfect fit for both datasets.

#### 2.2.4 History of Life Tables

The evolution of life tables spans centuries, beginning with John Graunt's work on age patterns in populations in his “Natural and Political Observations upon Bills of Mortality,”. From Graunt's conceptualizations, pioneers like Edmond Halley and Dr. Price Northampton contributed to the development of the first life tables. These early milestones laid the foundation for subsequent advancements, including Mine's Carlisle table and Dr. Farr's English Life Tables, constructed using census data.

Life tables stand as foundational tools in demography, offering invaluable insights into mortality patterns, longevity trends, and population dynamics across different epochs. From historical estimations to contemporary analyses, the evolution of life tables reflects the relentless pursuit of understanding and improving population health outcomes. By exploring various sets of life tables spanning from the mid-19th century to the present day, we gain a comprehensive understanding of societal changes, healthcare advancements, and demographic shifts that have shaped the trajectory of human life expectancy.

The origins of life tables in the United States can be traced back to the mid-19th century. Michael R. Haines' seminal work, published in 1994, presents three sets of estimated life tables for the U.S. during the period of 1850-1900. These tables provide crucial insights into mortality patterns and survival probabilities by age, sex, and race during this transformative era. Utilizing models such as the Brass two-parameter logit model and the Coale and Demeny West Model, Haines' work laid the groundwork for understanding mortality dynamics in the United States during this pivotal period.(Haines,1994)

In contemporary times, advancements in data collection and analysis have facilitated the development of robust life tables that offer a nuanced understanding of population health dynamics. The National Center for Health Statistics (NCHS) plays a central role in disseminating the most up-to-date life tables for the U.S. The 2021 life tables, adjusted for race and ethnicity misclassification, provide essential information on life expectancy, age-specific death rates, and survivorship probabilities (2022). Moreover, state-specific life tables offer localized perspectives on longevity disparities, enabling targeted interventions to address health inequities across diverse populations.

Over the years, life expectancy in the United States has witnessed a remarkable upward trajectory. From 1950 to 2024, life expectancy has steadily increased, reflecting significant advancements in healthcare, public health initiatives, and societal well-being. Factors such as medical innovations, preventive healthcare measures, and lifestyle changes have contributed to this improvement, underscoring the transformative impact of healthcare interventions on population health outcomes.(1950)

Projections of mortality patterns offer valuable insights into future demographic trends and inform long-term policy planning. The U.S. Census Bureau explores projected differences in life expectancy across various demographic groups, providing policymakers with valuable insights into future population health dynamics. A report by Lauren Medina, Shannon Sabo, and Jonathan Vespa gives projections of life expectancy from 2017 to 2060 and explores projected differences in mortality for men and women and for different race and Hispanic origin groups in the United States(2020). By anticipating demographic shifts and emerging challenges, stakeholders can develop proactive measures to address health disparities and promote equitable access to healthcare services, thereby fostering healthier and more resilient communities.

The evolution of life tables from historical estimations to contemporary analyses reflects a continuous quest to understand and improve population health outcomes. By leveraging empirical data, advanced statistical models, and interdisciplinary research, life tables serve as indispensable tools for policymakers, researchers, and healthcare professionals in navigating complex demographic trends and formulating evidence-based interventions to enhance societal well-being.

### 2.3 Research gap

Despite existing research on mortality modeling and comparative analysis, a notable research gap exists in comprehensively assessing mortality trends across different cohorts using advanced modeling techniques. While previous studies have examined mortality patterns within specific cohorts or utilized single models for projection, there is a lack of comparative analyses between cohorts from distinct time periods using multiple models. This research gap highlights the need for a study that systematically compares mortality trends for cohorts such as the 1981 and 2002 cohorts, which represent significant segments of the active population, using both the Cairns Blake Dowd and Renshaw Haberman models. By addressing this gap, the study aims to provide valuable insights into temporal variations in mortality patterns, the effectiveness of different modeling approaches, and their implications for actuarial practice. Such an analysis will contribute to advancing knowledge in mortality forecasting and support evidence-based decision-making in diverse domains.

# CHAPTER THREE

## METHODOLOGY

### 3.1 Summary

In this project , our aim was to forecast the male and female mortality rates of the US population using data obtained from the Human Mortality Database, focusing on the Renshaw Haberman and Cairns Blake Dowd model. Subsequently we created cohort life tables, one utilizing the projected mortality rates from the Renshaw Haberman model while the other from projected mortality rates from the CBD model. We then proceeded to plot and compare the mortality trends based on the predicted mortality rates of both models. To achieve this, we employed the secondary data incorporating certain features, preprocessing steps, validation of models, predicting of the mortality rates using the models, construction of cohort life tables for both male and females and a comparative analysis of the mortality trends.

### 3.2 Modelling Framework/Preliminaries

#### 3.2.1 Mortality Trends

: The probability of a person aged exactly x dying within the next t year.

= , the life expectancy of a person aged x

= : proportion of individuals in a population

that survives to different ages.

: Longevity Risk, the risk that members live longer than they are currently expected

#### 3.2.2 Model Selection and Justification

The selection of the Renshaw-Haberman and Cairns-Blake-Dowd (CBD) models for evaluation in mortality rate prediction was driven by several key factors. In this study, we used the modified Lee-Carter model, and Renshaw-Haberman model(2006), which take into account cohort effects. Both the Renshaw-Haberman and CBD models are widely recognized and their widespread adoption in academic research and practical applications makes them relevant for comparative evaluation. The Renshaw-Haberman model, utilizing singular value decomposition to capture general trends and age-specific patterns, and the CBD model, incorporating additional components for flexibility, offer methodological diversity enabling a comprehensive assessment of their respective strengths and weaknesses in mortality prediction.

Additionally, both the CBD and Renshaw-Haberman models are commended for their simplicity and robustness. Comparative analysis with the Renshaw-Haberman model allows for an understanding of how advancements in modeling, as seen in the CBD model, contribute to improved accuracy or other desirable attributes. The well-documented methodologies and implementation in widely used statistical software for both the Renshaw-Haberman and CBD models ensure accessibility, facilitating a standardized evaluation process for researchers and practitioners. The Renshaw-Haberman model contains the interaction component, while the CBD model does not contain the interaction component but includes two time-specific components.

#### 3.2.3 Data Collection

It’s important to note that the quality of our data will be assessed by checking for missing values, outliers, and inconsistencies. Any limitations or biases in the data that could affect our results, such as under-reporting of deaths or inaccuracies in age reporting were addressed. The Human Mortality database provides comprehensive and detailed mortality and population data for numerous countries around the world.The USA data obtained from the Human Mortality Database (HMD) provides a comprehensive information on mortality and population statistics of the United States over multiple decades (1933-2021). The data is provided for both males and females as well as total with their deaths and mortality rates.

**3.2.4 Software and Tools**

R and Power BI were used. Different packages in R were used to clean data, model testing and validation, and make the predictions. Power BI was employed for data visualizations.

### 3.3 Objective 1: To estimate mortality rates using the CBD and Renshaw-Haberman models from their current age in the year 2022 up to age 80.

#### 3.3.1 Renshaw-Haberman Model

The Renshaw-Haberman model is a stochastic statistical method for forecasting mortality rates based on historical data. In this study, we will use the modified Lee-Carter model(Renshaw-Haberman), which takes into account cohort effects.

Its assumptions are:

* The logarithm of the crude death rate at a given age and time can be decomposed into three components: an age-specific component, a time-specific component, an interaction component, and a cohort-specific component.
* The age-specific component captures the general shape of the mortality curve across different ages and is assumed to be constant over time.
* The time-specific component captures the overall level of mortality and its trend over time and is assumed to follow a random walk with drift.
* The cohort-specific component captures the cohort effects of individuals aged *x* years born in the year *t-x* and assumes that people born in the same generation will experience the same trend in mortality.

The model can be expressed as

(1)

where:

* is the central death rate at age x and time t
* is the age-specific component.
* is the age-specific sensitivity to the time component.
* is the time-specific component.
* is the cohort-specific component.
* is the error term.

The model parameters are estimated by singular value decomposition (SVD), a technique that minimizes the sum of squared errors. The model can then forecast future mortality rates by extrapolating the time-specific component and applying the estimated age-specific and interaction components.

Singular Value Decomposition (SVD) is a matrix factorization technique used in linear algebra and numerical analysis. It breaks down a matrix into three constituent matrices, enabling a deeper understanding of the data's structure and aiding in various mathematical computations and data analyses.

To estimate the parameters of the Renshaw-Haberman model, we first transform the age-specific mortality rates using the logarithm . Then create a matrix *M* where each row corresponds to an age group, each column corresponds to a time period, and the entries are the transformed mortality rates. Apply SVD to the matrix *M* given by;

*M*=U (2)

Where;

U is the left singular vectors matrix.

(sigma) is a diagonal matrix of singular values.

is the transpose of the right singular vectors matrix.

The values of , and are extracted from the matrices where;

The values of are related to the first column of U.

The values of are related to the first column of V.

The values of related to the first singular value and the corresponding columns of U and .

= (3)

= (4)

= \* (5)

Here,and denote the first column of matrices U and respectively, is the first singular value and *T* is the number of time periods.

#### 

#### 3.3.2 Cairns-Blake-Dowd Model

The Cairns-Blake-Dowd (CBD) model is a two-factor stochastic method.

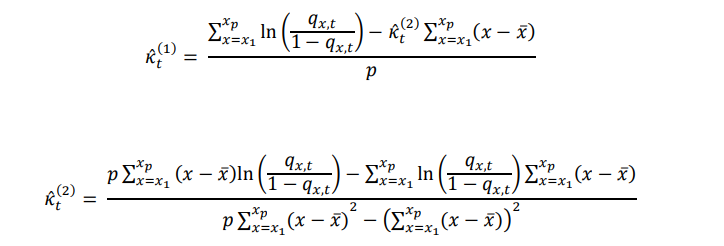
The Cairns-Blake-Dowd (CBD) model using the logit of the mortality rate can be expressed as follows:

+ (6)

where:

* is the mortality rate at age x and time t.
* and are the time-specific components.
* x̄ is the average age in the data.
* is the cohort-specific component.
* is the error term.

This model is either a log-Poisson or a log-negative-binomial version of the CBD model. The model parameters are estimated by the ordinary least squares method. Ordinary Least Squares (OLS) is a method used in statistical regression analysis to estimate the parameters of a linear regression model. The goal of linear regression is to find the best-fitting straight line through a set of data points that minimizes the sum of the squared errors. The following formulas will be used;



The Cairns-Blake-Dowd (CBD) model has several assumptions:

* The model assumes that the age effects are simple and allow different improvements at different ages at different times.
* The model assumes that there is a perfect correlation across ages.
* The model accounts for cohort effects, which are the influences of shared experiences and characteristics of people born in the same year or period.
* The model assumes that ,at older ages say from 50 ,the mortality behaves more or less linear

Variables involved in the models:

**(i) Age-specific component ()**

The age-specific component refers to the baseline level of mortality for a particular age group or cohort in the absence of the effects of the time trend, cohort effects, or other factors. It is the general shape of mortality by age.

**(ii) Time-specific component**

The time-specific component has different dynamics for the two models. In Renshaw-Haberman, , is the time index representing the level of mortality at time t.

In CBD,, is the intercept of the model. It affects every age in the same way and it represents the level of mortality at time t. represents the slope of the model, every age is differently affected by this parameter.

**(iii) Cohort Effects ( )**

The cohort effect represents the impact of shared experiences, environmental factors, and other cohort-specific influences on mortality rates.

**iv)Interaction component**

The interaction component is the product and it represents the impact of the overall time trend on the age-specific mortality rates . It describes mortality dynamics over age and time. The values determine how sensitive each age group is to the overall trend in mortality. describes the extent to which mortality at age x changes given the overall temporal change in the general level of mortality: greater values of are associated with faster mortality change.

#### 3.3.3 Assumption on mortality

In this study, we will assume that mortality was not affected by the Covid-19 pandemic.

#### 3.3.4 Model Validation

To validate our model, we plan to use out-of-sample validation where some of our data will be held out for testing the model’s predictions.

#### 3.3.5 Performance evaluation of the models

Evaluating the performance of a predictive model is essential to gauge its accuracy and reliability in making predictions. By employing various metrics, we can objectively measure how well the models perform in predicting outcomes based on the provided data. These metrics allow us to understand the extent to which the model's predictions align with the actual observed values. In this analysis, we'll be using RMSE and MAE, two widely accepted measures in the field of predictive modeling, to quantify the accuracy of our predictions and understand the average magnitude of errors between predicted and actual.

RMSE measures the average difference between values predicted by a model and the actual values. It provides an estimation of how well the model is able to predict the target value (accuracy). The lower the value of the RMSE, the better the model is.

It is given by:

**RMSE**= (9)

Where;

y = actual mortality rates

= Predicted mortality rates

The Mean Absolute Error (MAE) is a metric used to measure the average magnitude of errors between predicted and actual values. It assesses the accuracy of a predictive model. Lower MAE values indicate better performance, but the interpretation of ‘how good’ is often problem-specific. It's also common to use MAE in conjunction with other metrics to get a more comprehensive understanding of a model's performance.

It is given by:

**MAE**= (10)

Where;

y = actual mortality rates

= Predicted mortality rates

### 3.4 Objective 2: To construct cohort life tables, using the Renshaw-Haberman and CBD model.

Using the collected data and predicted mortality rates we will construct the male and female cohort life tables for the two models.

The values predicted from the Renshaw-Haberman model are the values of , hence we need to calculate the values of from the relationship between the two.

The death rate,), and the mortality rate, ), are typically very close to one another in value. With a simple assumption, we can formalize this relationship more precisely:

**Assumption:** For integers t and x, and for all, that is, the force of mortality remains constant over each year of integer age and over each calendar year. This implies that:

a. (11)  
 b. (12)

Relationship (a) is often used in the analysis of death rate data. Relationship (b) is useful in the analysis of parametric models for mortality that are formulated in terms of . The assumption does not normally hold exactly, but the resulting relationship between ) and) is generally felt to provide an accurate approximation.

#### 3.4.1 Assumptions of our Life Tables

1. The only form of exit is death.
2. Assumes a closed cohort where births, deaths, immigration, and emigration are not considered. This assumption simplifies calculations and analysis.
3. Individuals within the cohort are assumed to have the same mortality characteristics and variations within the cohort are not explicitly considered.

Our life tables will contain the following variables;

x: age

: the “survivor-ship function”: the number of persons alive at age x.  
 : number of deaths in the interval (x, x + 1) for persons alive at age x, computed as (13)

: the probability of a person aged exactly x will die before reaching age x+1

: the mortality rate or the central death rate at age x.

: total number of person-years lived by the cohort from age x to x + 1. This is the sum of the years lived by the persons who survive the interval, and the dx persons who die during the interval.

(14)  
 =: total number of person-years lived by the cohort from age x until all members of the cohort have died.  
 : the life expectancy of persons alive at age x, computed as

= . (15)

We will use R to generate the tables.

### 3.5 Objective 3: To compare the mortality trends based on the two models

#### 3.5.1 Comparison of mortality trends

The trend analysis involves analyzing; time trends of mortality rate, overall mortality rate, survivorship curves, cumulative death rates, gender differences, probability of death, crude death rate, and life expectancies against time.

For comparison, we needed the life expectancy values ( ), mortality rates (, and the crude death rate() from the life tables. We plotted each of these values against time (ages or years) to get a visual of how the mortality trend of the cohorts move as they age depending on the predictions of the two models. We conducted a comprehensive analysis of these trends to identify and explain any differences and similarities that occurred.

# CHAPTER FOUR

## DATA ANALYSIS

### 4.1 Data

#### 4.1.1 Data collection

The data used for this study was secondary data for the United States of America from the Human Mortality Database for the years 1933 to 2021. The data included yearly information about the mortality rates, exposures, number of deaths and number of births for both males and females. The data for ages beyond 80 was not considered as the data may not be very reliable because the human mortality database has a protocol that harmonizes the data after age 80. The data was used to fit the models, forecast mortality rates, and construct life tables.

### 4. 2 Exploratory Data Analysis

The following figures report for the USA Population the pattern of logarithm of death rates according to age and time for males and females. Different colors indicate different years (most recent ones in violet, earliest in red). Several behavioral patterns are shown respectively for male, and female as the different cohorts age. One pattern being that generally, the mortality declines as the years progress. Additionally, there is a dip in mortality at younger ages showing that overally, mortality improved faster at younger ages compared to the older ages which can be attributed to the increased targeted healthcare interventions such as prenatal care and child immunization.

Figure 1: Death rates against age


Figure 1: Death Rates against age

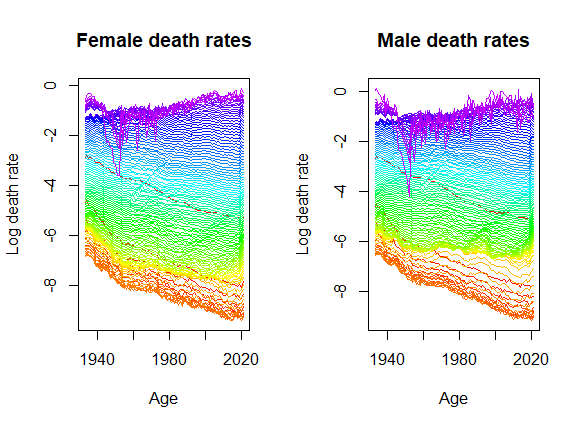


Figure 2:Death Rates against age over different years

### 4.3 Objective 1: To estimate mortality rates using the CBD and Renshaw-Haberman models.

#### 4.3.1: Fitting the Renshaw Haberman model

The model can be expressed as equation (1). The model was fit separately for males and females from age 0 to 80.

**RH female parameters**

is the age specific component that shows the general trend of mortality for different ages. The initial ages have high mortality rates due to infant mortality that later on decreases due to improved immune systems for the younger ages between 7 to 11. There was a slight increase in mortality for the ages about 12 to about 28 due to factors such as substance abuse, suicidal cases as well as violence.

is the age specific component sensitive to time, the trend of this parameter shows the volatility of the age specific component at different ages over time. For the ages 0 to about 5, there was sharp increase implying that the mortality is highly bound to shift due to infant mortality. Between age 20 and 40, there was a noticeable increase then decrease in which is influenced by factors such as lifestyle changes, occupation and maternal mortality. For the older ages from the age 40, there’s decreased sensitivity to the changes in mortality due to being accustomed to the factors (a large proportion of the women are past their bearing ages) initially experienced in the ages 20 to 40.

was initially high for the 1933 to 1940 period due to the impact of world war *II*as well as the great depression which had adverse effects on mortality. From the 1940 to 1960 period the mortality lowers and stabilizes due to the booming after the war period. The mortality increased gradually from the 1980 period due to outbreak of pandemics such as HIV and AIDS in the 1980’s and in 2020 due to the Covid-19 outbreak.

Lastly, the cohort effect has a decreasing trend over time to show that with time the different cohorts experience mortality improvements, mortality decreases. Over time, mortality has dropped for the various cohorts due to advancements in both the health sector and lifestyle.

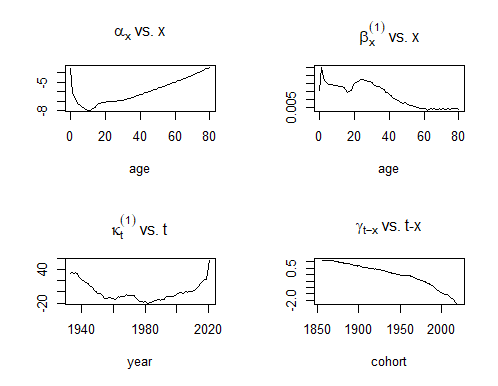


Figure 3:RH female Parameters

* is the age-specific component.
* is the age-specific sensitivity to the time component.
* is the time-specific component.
* is the cohort-specific component.

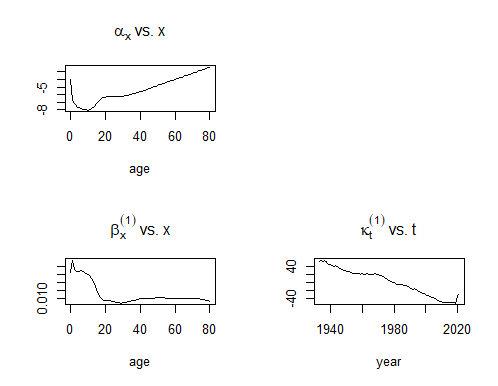
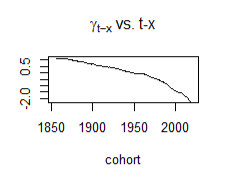
**RH males parameters**

is the age specific component that provided the general trend of mortality for different ages. The initial ages have high mortality rates due to infant mortality that later on decreased due to improved immune systems for the younger ages between 5 to 11. There was slight increase in mortality for the ages about 12 to about 20 due to factors such as substance abuse, suicidal cases and violence where men at these ages tend to engage in risky situations. is the age specific component sensitive to time, the trend of this parameter shows the volatility of the age specific component at different ages over time where for the ages 0 to about 5, there was sharp increase implying that the mortality is highly bound to shift due to infant mortality. For the ages after 5 to about the age of 20 the sensitivity decreased. For the ages above 20, the male population is less sensitive due to lowered values of the age specific component sensitive to time.

was initially high for the 1933 to 1940 period due to the impact of World War IIwhich had adverse effects on mortality. From the 1940 to about 1990's the mortality decreased gradually. The mortality increased a bit for the 1980 period due to outbreak of pandemics such as HIV and AIDS in the 1980’s and spiked in 2020 due to the Covid-19 outbreak.

Lastly, the cohort effect has a decreasing trend over time to show that with time the different cohorts experience mortality improvements, mortality decreases. With improvements in the health sector as well as improved lifestyle, there’s decreased mortality for the different cohorts over time.

Figure 4: RH male parameters

****

* is the age-specific component.
* is the age-specific sensitivity to the time component.
* is the time-specific component.
* is the cohort-specific component.

#### 

#### 4.3.2: Fitting the CBD model

The model was fitted from age 0 to age 80.

**CBD parameters:**

In the CBD model, the male and female parameters are seen to follow a similar pattern. The parameter shows the level of mortality over time after a logit transformation. In both graphs it can be seen that the parameter ,as expected, has a bell shape which in this case indicates that in recent years there has been an improvement in the overall mortality which can be attributed to advancements in medicine and healthcare, public health initiatives, technological innovations and improved standards of living, . The second CBD mortality index, , represents the slope of the logit-transformed mortality curve. An increase in , that is, an increase in the steepness of the logit-transformed mortality curve, means that mortality (in logit scale) at younger ages (below the mean age ) improves more rapidly than at older ages (above the mean age ).

The faster improvement in mortality rates at younger ages compared to older ages can be attributed to various factors. Advances in pediatric medicine, including neonatal care and childhood immunization programs, have notably reduced infant and child mortality. Early interventions like prenatal care and childhood health initiatives help detect and address health issues early on. Socioeconomic factors also play a role, with improved access to healthcare and education narrowing mortality gaps. Younger individuals may respond better to health promotion efforts and lifestyle changes, contributing to lower mortality from preventable conditions. Additionally, targeted interventions in maternal and child health services further accelerate progress in reducing mortality rates among younger age groups.

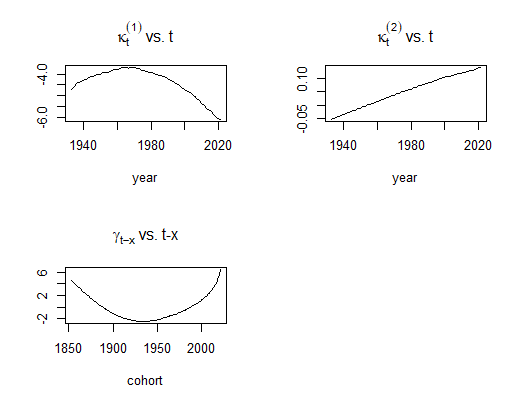
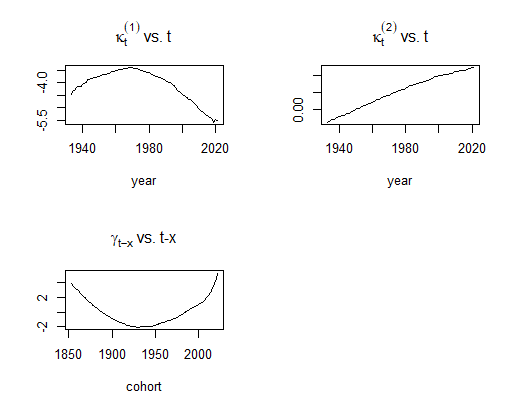


Figure 5: CBD female parameters

**CBD male parameters**

Figure 6: CBD male parameters

****

#### 4.3.3: Goodness of fit

Goodness-of-fit tests are essential when comparing the accuracy of two predictive models because they provide statistical measures to assess how well each model fits the observed data. These tests evaluate whether the observed data matches the expected outcomes according to the predictions of the models. The two widely accepted measures in the field of predictive modeling, to quantify the accuracy of our predictions and understand the average magnitude of errors between predicted and actual are the Root Mean Squared Error (RMSE) and Mean Absolute error (MAE).

##### **RMSE**

It is expressed in the same units as the target variable. A smaller RMSE indicates better performance. The closer RMSE is to 0, the better the model's predictions align with the actual values. The RMSE is expressed in equation (9).

Table 1:RMSE Estimates

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **MODELS** | |
| **Gender** | **Cohort** | **RH** | **CBD** |
| *Female* | 1981 | 0.0009505 | 0.0015531 |
| 2002 | 0.0004488 | 0.0013369 |
| *Male* | 1981 | 0.0002668 | 0.0019189 |
| 2002 | 0.0002535 | 0.0016520 |

In both the female and male subsets, the RH (Renshaw-Haberman) model demonstrates lower Root Mean Squared Error (RMSE) values compared to the CBD (Cains-Blake-Dowd) model for both 1981 and 2002. The RH model consistently exhibited superior performance over the CBD model across all subsets and years. Therefore, based on RMSE findings, the RH model is favored over the CBD model for both female and male subsets in both 1981 and 2002.

##### **Mean Absolute Error:**

It’s a measure of absolute difference between the predicted and observed values. It’s expressed in the same units as the original data hence easy to interpret. The MAE is expressed in equation (10).

Table 2:MAE Estimates

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **MODELS** | |
| **Gender** | **Cohort** | **RH** | **CBD** |
| *Female* | 1981 | 0.01507 | 0.1951 |
| 2002 | 0.008928 | 0.3041 |
| *Male* | 1981 | 0.01507 | 0.1962 |
| 2002 | 0.008928 | 0.2846 |

In both female and male subsets, the RH model demonstrates lower Mean Absolute Error (MAE) values compared to the CBD model for both 1981 and 2002. Lower MAE values indicate better model performance, with the RH model consistently showing superior performance over the CBD model across all subsets and years. Therefore, based on MAE findings, the RH model is favored over the CBD model for both female and male subsets in both 1981 and 2002.

**Model Comparison**

**Deviance**

Deviance residuals are a measure of the goodness of fit of the model and are particularly useful in assessing the adequacy of GLMs. Deviance is a measure of the difference between the model's predicted probabilities and the observed outcomes. It quantifies the lack of fit of the model to the data, similar to how residuals in linear regression quantify the lack of fit of the model to continuous outcome variables.

In a well-fitted model, deviance residuals should exhibit random scatter around zero. If there are systematic patterns or trends in the deviance residuals, it suggests that the model fails to capture some aspect of the data, potentially indicating problems with model specification or missing predictor variables.

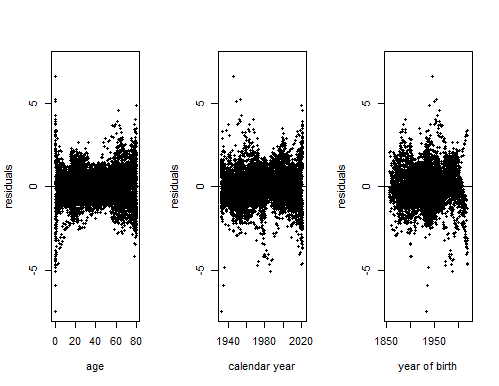
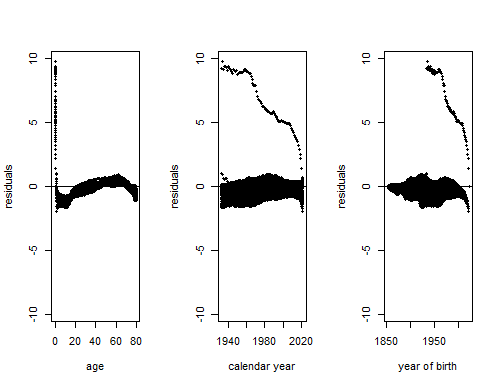
**Female residuals**

Figure 7:RH model Residuals Females

**RH model**

**CBD Model**

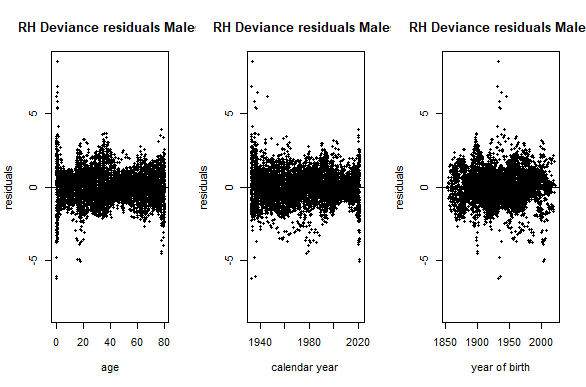
Figure 8: CBD model Residuals Females

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**Male Residuals**

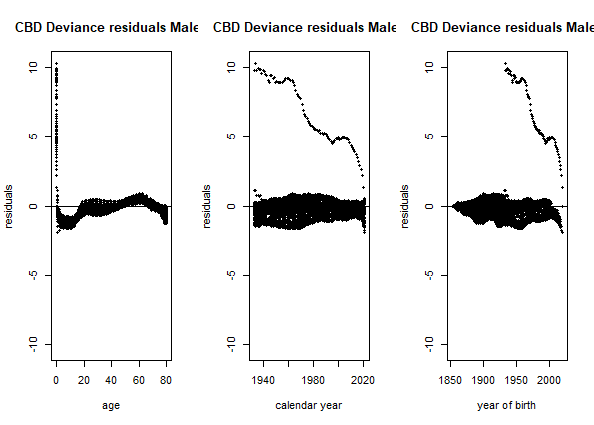
**RH**

Figure 9: RH model Residuals Males

****

**CBD**

Figure 10: CBD Residuals Males

****

Residuals by Age:

Residuals in the CBD model for both males and females exhibit a slightly curved pattern, indicating potential nonlinearity or heteroscedasticity. Nonlinearity indicates that the relationship between the predictor variables and the response variable may not be adequately captured by the linear model. Heteroscedasticity refers to the situation where the variability of the residual’s changes across different levels of the predictor variables, violating the assumption of constant variance in a linear regression model. Additionally, a clear pattern of outliers is evident in the scatterplot of deviance residuals for CBD. The scatterplot for the residuals in RH shows a seemingly random distribution of deviance residuals around zero, with no apparent systematic patterns or trends for both the males and the females. Any remaining deviations are likely due to random fluctuations.

Residuals by calendar year:

Residuals in both RH and CBD cluster tightly around zero, but a notable upward trend in positive residuals emerges around the 1980s in both genders, due to methodological changes or societal shifts. However, in RH, the residuals appear to be scattered evenly across the range of years suggesting uniform variability in model predictions while in CBD, there is a clear pattern of outliers.

Residuals by year of birth:

Residuals in the CBD model for both males and females exhibit a tight cluster around zero but a slight negative deviation towards the end indicating that the model may underestimate mortality for cohorts that are further away. The scatterplot for the residuals in RH shows a seemingly random distribution of deviance residuals around zero, with no apparent systematic patterns or trends for both the males and the females. CBD also has a significant and clear pattern of observations as outliers.

For the RH model, the residuals show a relatively stable pattern across different age groups with minor deviations around zero residual value, indicating a reasonable fit. The upward trend in positive residuals around the 1980s suggests some discrepancies during that period.

On the other hand, CBD model residuals also exhibit a dense cluster around zero, indicating a good fit initially. However, there is a noticeable increase in positive residuals around the early 1900s.This suggests that the model may not capture the full variation in the data for individuals born during later years. In summary, the Renshaw-Haberman model appears to perform better in terms of stability across age group although the CBD model shows more variability, especially for specific birth cohorts.

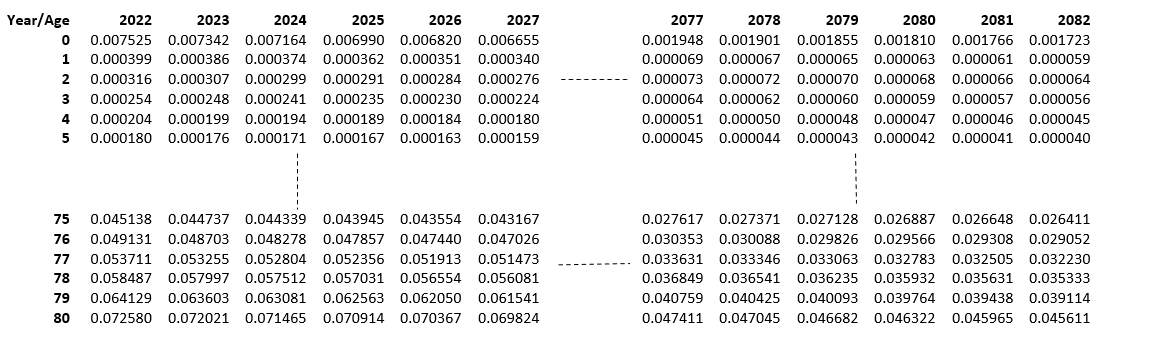
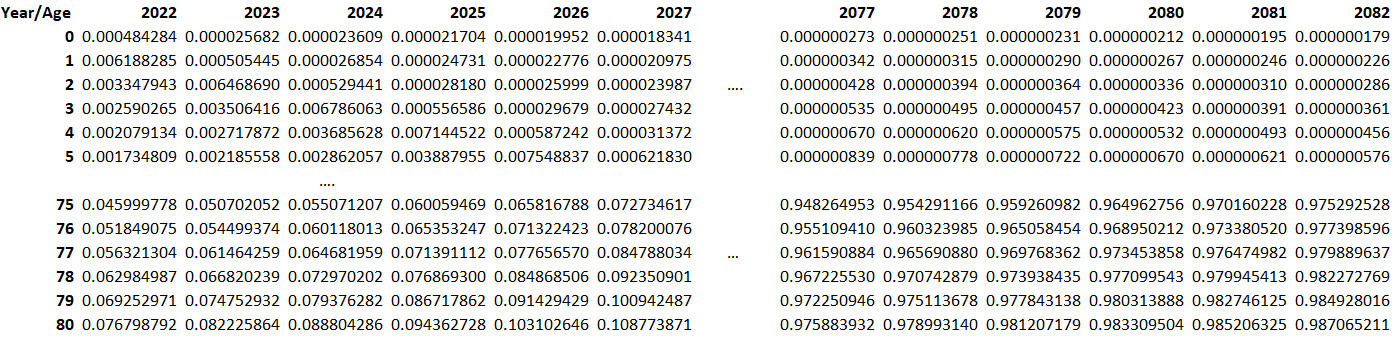
The two models were then used to forecast the mortality rates from the year 2022 to 2082(till age 80 of the chosen cohorts).

Table 4:Male mortality forecasts from RH model

Table 3: Female mortality forecasts from RH model



### 4.4 Objective 2: To construct cohort life tables, using the mortality rates from the preferred model

The life tables for the 1981 and 2002 cohorts were generated using the mortality rates for the Renshaw-Haberman model. From the tables, it is clear that different generations experience mortality differently causing differences in their life expectancies. The 2002 cohort has a higher life expectancy to the 1981 cohort with a noticeable pattern of declining mortality. Additionally, the life expectancies of males and females are different with females having a higher life expectancies. The life tables for the two cohorts are illustrated below;

Table 5: 1981 Female Life Table Generated using Mortality Rates of RH

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 0 | 1768966 | 18857 | 0.01069 | 0.010630 | 1759537.54 | 132387277.76 | 74.84 | 1.0000 |
| 1 | 1750109 | 1383 | 0.00081 | 0.000812 | 1749417.51 | 130627740.22 | 74.64 | 0.9893 |
| 2 | 1748726 | 860 | 0.00050 | 0.000504 | 1748295.94 | 128878322.71 | 73.70 | 0.9886 |
| 3 | 1747866 | 614 | 0.00036 | 0.000356 | 1747558.96 | 127130026.78 | 72.73 | 0.9881 |
| 4 | 1747252 | 453 | 0.00026 | 0.000260 | 1747025.29 | 125382467.82 | 71.76 | 0.9877 |
| 5 | 1746799 | 389 | 0.00022 | 0.000223 | 1746604.20 | 123635442.53 | 70.78 | 0.9875 |
| 6 | 1746410 | 371 | 0.00021 | 0.000213 | 1746224.11 | 121888838.33 | 69.79 | 0.9872 |
| 7 | 1746038 | 346 | 1.97E-04 | 0.000197 | 1745865.56 | 120142614.22 | 68.81 | 0.9870 |
| 8 | 1745693 | 325 | 0.00019 | 0.000192 | 1745530.41 | 118396748.65 | 67.82 | 0.9868 |
| 9 | 1745368 | 301 | 1.67E-04 | 0.000167 | 1745217.65 | 116651218.24 | 66.83 | 0.9867 |
| 10 | 1745067 | 288 | 1.58E-04 | 0.000158 | 1744923.33 | 114906000.59 | 65.85 | 0.9865 |
| 11 | 1744779 | 274 | 1.51E-04 | 0.000151 | 1744642.53 | 113161077.27 | 64.86 | 0.9863 |
| 12 | 1744506 | 308 | 0.00017 | 0.000168 | 1744351.77 | 111416434.74 | 63.87 | 0.9862 |
| 13 | 1744198 | 358 | 0.00019 | 0.000193 | 1744019.01 | 109672082.96 | 62.88 | 0.9860 |
| 14 | 1743840 | 487 | 0.00026 | 0.000260 | 1743596.61 | 107928063.95 | 61.89 | 0.9858 |
| 15 | 1743353 | 599 | 0.00032 | 0.000316 | 1743053.50 | 106184467.34 | 60.91 | 0.9855 |
| 16 | 1742754 | 801 | 0.00042 | 0.000420 | 1742353.60 | 104441413.84 | 59.93 | 0.9852 |
| 17 | 1741953 | 825 | 0.00043 | 0.000430 | 1741541.04 | 102699060.24 | 58.96 | 0.9847 |
| 18 | 1741129 | 906 | 0.00046 | 0.000464 | 1740675.92 | 100957519.20 | 57.98 | 0.9843 |
| 19 | 1740223 | 899 | 0.00045 | 0.000447 | 1739773.42 | 99216843.28 | 57.01 | 0.9838 |
| 20 | 1739324 | 899 | 0.00044 | 0.000444 | 1738874.24 | 97477069.86 | 56.04 | 0.9832 |
| 21 | 1738425 | 1017 | 0.00050 | 0.000498 | 1737916.27 | 95738195.61 | 55.07 | 0.9827 |
| 22 | 1737408 | 1037 | 0.00051 | 0.000506 | 1736889.55 | 94000279.35 | 54.10 | 0.9822 |
| 23 | 1736371 | 948 | 0.00046 | 0.000460 | 1735897.46 | 92263389.79 | 53.14 | 0.9816 |
| 24 | 1735424 | 1036 | 0.00050 | 0.000499 | 1734905.88 | 90527492.33 | 52.16 | 0.9810 |
| 25 | 1734388 | 1093 | 0.00052 | 0.000519 | 1733841.61 | 88792586.45 | 51.20 | 0.9805 |
| 26 | 1733295 | 1171 | 0.00056 | 0.000557 | 1732709.64 | 87058744.84 | 50.23 | 0.9798 |
| 27 | 1732124 | 1149 | 0.00054 | 0.000539 | 1731549.75 | 85326035.20 | 49.26 | 0.9792 |
| 28 | 1730975 | 1294 | 0.00061 | 0.000611 | 1730328.10 | 83594485.45 | 48.29 | 0.9785 |
| 29 | 1729681 | 1320 | 0.00063 | 0.000628 | 1729020.86 | 81864157.35 | 47.33 | 0.9778 |
| 30 | 1728361 | 1416 | 0.00067 | 0.000667 | 1727652.63 | 80135136.49 | 46.36 | 0.9770 |
| 31 | 1726944 | 1484 | 0.00070 | 0.000696 | 1726202.24 | 78407483.86 | 45.40 | 0.9762 |
| 32 | 1725460 | 1614 | 0.00075 | 0.000754 | 1724653.05 | 76681281.62 | 44.44 | 0.9754 |
| 33 | 1723846 | 1779 | 0.00083 | 0.000828 | 1722956.49 | 74956628.57 | 43.48 | 0.9745 |
| 34 | 1722067 | 1995 | 0.00093 | 0.000925 | 1721069.33 | 73233672.08 | 42.53 | 0.9735 |
| 35 | 1720072 | 2316 | 0.00107 | 0.001069 | 1718913.68 | 71512602.75 | 41.58 | 0.9724 |
| 36 | 1717756 | 2463 | 0.00114 | 0.001134 | 1716524.39 | 69793689.07 | 40.63 | 0.9711 |
| 37 | 1715293 | 2585 | 0.00119 | 0.001189 | 1714000.59 | 68077164.68 | 39.69 | 0.9697 |
| 38 | 1712708 | 2896 | 0.00133 | 0.001327 | 1711260.06 | 66363164.09 | 38.75 | 0.9682 |
| 39 | 1709812 | 3552 | 0.00162 | 0.001619 | 1708036.03 | 64651904.03 | 37.81 | 0.9666 |
| 40 | 1706260 | 4475 | 0.00205 | 0.002044 | 1704022.64 | 62943868.00 | 36.89 | 0.9646 |
| Continued on next page | | | | |  | | | |

*Table 5 Continued from previous page*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 41 | 1701785 | 3558 | 0.00209 | 0.002091 | 1700006.22 | 61239845.36 | 35.99 | 0.9620 |
| 42 | 1698227 | 3566 | 0.00210 | 0.002100 | 1696444.20 | 59539839.14 | 35.06 | 0.9600 |
| 43 | 1694661 | 3658 | 0.00216 | 0.002159 | 1692832.14 | 57843394.93 | 34.13 | 0.9580 |
| 44 | 1691003 | 3792 | 0.00225 | 0.002242 | 1689107.01 | 56150562.80 | 33.21 | 0.9559 |
| 45 | 1687211 | 3906 | 0.00232 | 0.002315 | 1685257.78 | 54461455.79 | 32.28 | 0.9538 |
| 46 | 1683305 | 4084 | 0.00243 | 0.002426 | 1681262.67 | 52776198.01 | 31.35 | 0.9516 |
| 47 | 1679221 | 4230 | 0.00252 | 0.002519 | 1677105.95 | 51094935.34 | 30.43 | 0.9493 |
| 48 | 1674991 | 4369 | 0.00261 | 0.002609 | 1672806.44 | 49417829.39 | 29.50 | 0.9469 |
| 49 | 1670622 | 4698 | 0.00282 | 0.002812 | 1668272.82 | 47745022.96 | 28.58 | 0.9444 |
| 50 | 1665924 | 4899 | 0.00295 | 0.002941 | 1663474.43 | 46076750.13 | 27.66 | 0.9418 |
| 51 | 1661025 | 5131 | 0.00309 | 0.003089 | 1658459.33 | 44413275.70 | 26.74 | 0.9390 |
| 52 | 1655894 | 5313 | 0.00321 | 0.003209 | 1653236.95 | 42754816.36 | 25.82 | 0.9361 |
| 53 | 1650580 | 5540 | 0.00336 | 0.003356 | 1647810.23 | 41101579.41 | 24.90 | 0.9331 |
| 54 | 1645040 | 5789 | 0.00353 | 0.003519 | 1642145.99 | 39453769.18 | 23.98 | 0.9299 |
| 55 | 1639252 | 6021 | 0.00368 | 0.003673 | 1636241.03 | 37811623.19 | 23.07 | 0.9267 |
| 56 | 1633230 | 6266 | 0.00384 | 0.003837 | 1630097.31 | 36175382.16 | 22.15 | 0.9233 |
| 57 | 1626964 | 6647 | 0.00409 | 0.004086 | 1623640.68 | 34545284.85 | 21.23 | 0.9197 |
| 58 | 1620317 | 6786 | 0.00420 | 0.004188 | 1616923.98 | 32921644.18 | 20.32 | 0.9160 |
| 59 | 1613531 | 7384 | 0.00459 | 0.004576 | 1609838.71 | 31304720.20 | 19.40 | 0.9121 |
| 60 | 1606147 | 7705 | 0.00481 | 0.004797 | 1602293.84 | 29694881.49 | 18.49 | 0.9080 |
| 61 | 1598441 | 8243 | 0.00517 | 0.005157 | 1594319.83 | 28092587.65 | 17.57 | 0.9036 |
| 62 | 1590199 | 8505 | 0.00536 | 0.005349 | 1585945.82 | 26498267.82 | 16.66 | 0.8989 |
| 63 | 1581693 | 9223 | 0.00585 | 0.005831 | 1577081.74 | 24912322.00 | 15.75 | 0.8941 |
| 64 | 1572470 | 9964 | 0.00636 | 0.006337 | 1567488.12 | 23335240.27 | 14.84 | 0.8889 |
| 65 | 1562506 | 10429 | 0.00670 | 0.006675 | 1557291.31 | 21767752.15 | 13.93 | 0.8833 |
| 66 | 1552077 | 11298 | 0.00731 | 0.007279 | 1546427.66 | 20210460.84 | 13.02 | 0.8774 |
| 67 | 1540779 | 12083 | 0.00787 | 0.007842 | 1534737.12 | 18664033.18 | 12.11 | 0.8710 |
| 68 | 1528696 | 12901 | 0.00848 | 0.008439 | 1522245.19 | 17129296.06 | 11.21 | 0.8642 |
| 69 | 1515795 | 14060 | 0.00932 | 0.009276 | 1508764.67 | 15607050.87 | 10.30 | 0.8569 |
| 70 | 1501735 | 14858 | 0.00994 | 0.009894 | 1494305.76 | 14098286.20 | 9.39 | 0.8489 |
| 71 | 1486877 | 16382 | 0.01108 | 0.011018 | 1478685.78 | 12603980.44 | 8.48 | 0.8405 |
| 72 | 1470495 | 17171 | 0.01175 | 0.011677 | 1461908.99 | 11125294.66 | 7.57 | 0.8313 |
| 73 | 1453323 | 18500 | 0.01281 | 0.012729 | 1444073.41 | 9663385.67 | 6.65 | 0.8216 |
| 74 | 1434824 | 20222 | 0.01419 | 0.014094 | 1424712.51 | 8219312.26 | 5.73 | 0.8111 |
| 75 | 1414601 | 21270 | 0.01515 | 0.015036 | 1403966.65 | 6794599.75 | 4.80 | 0.7997 |
| 76 | 1393332 | 22953 | 0.01661 | 0.016474 | 1381855.07 | 5390633.10 | 3.87 | 0.7877 |
| 77 | 1370378 | 24982 | 0.01840 | 0.018230 | 1357887.49 | 4008778.02 | 2.93 | 0.7747 |
| 78 | 1345397 | 26881 | 0.02018 | 0.019980 | 1331956.32 | 2650890.54 | 1.97 | 0.7606 |
| 79 | 1318516 | 28598 | 0.02193 | 0.021689 | 1304217.14 | 1318934.22 | 1.00 | 0.7454 |
| 80 | 1289918 | 29434 | 0.02308 | 0.022819 | 14717.08 | 14717.08 | 0.01 | 0.7292 |

Table 6:1981 Male Life Table Generated using Mortality Rates of RH

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 0 | 1860272 | 24462 | 0.01324 | 0.013150 | 1848041 | 131626841 | 70.76 | 1.0000 |
| 1 | 1835810 | 1741 | 0.00098 | 0.000970 | 1834939 | 129778800 | 70.69 | 0.9869 |
| 2 | 1834069 | 1171 | 0.00066 | 0.000660 | 1833484 | 127943860 | 69.76 | 0.9859 |
| 3 | 1832899 | 784 | 0.00044 | 0.000430 | 1832506 | 126110376 | 68.80 | 0.9853 |
| 4 | 1832114 | 719 | 0.00039 | 0.000390 | 1831755 | 124277870 | 67.83 | 0.9849 |
| 5 | 1831395 | 564 | 0.00031 | 0.000310 | 1831113 | 122446115 | 66.86 | 0.9845 |
| 6 | 1830831 | 584 | 0.00032 | 0.000320 | 1830539 | 120615002 | 65.88 | 0.9842 |
| 7 | 1830247 | 476 | 0.00026 | 0.000260 | 1830009 | 118784463 | 64.90 | 0.9839 |
| 8 | 1829771 | 498 | 0.00028 | 0.000280 | 1829522 | 116954454 | 63.92 | 0.9836 |
| 9 | 1829273 | 422 | 0.00022 | 0.000220 | 1829062 | 115124932 | 62.93 | 0.9833 |
| 10 | 1828851 | 451 | 0.00024 | 0.000230 | 1828626 | 113295870 | 61.95 | 0.9831 |
| 11 | 1828401 | 432 | 0.00023 | 0.000230 | 1828184 | 111467244 | 60.96 | 0.9829 |
| 12 | 1827968 | 520 | 0.00027 | 0.000270 | 1827708 | 109639059 | 59.98 | 0.9826 |
| 13 | 1827448 | 673 | 0.00035 | 0.000350 | 1827111 | 107811351 | 59.00 | 0.9824 |
| 14 | 1826775 | 980 | 0.00050 | 0.000500 | 1826285 | 105984240 | 58.02 | 0.9820 |
| 15 | 1825794 | 1187 | 0.00059 | 0.000590 | 1825201 | 104157955 | 57.05 | 0.9815 |
| 16 | 1824607 | 1680 | 0.00083 | 0.000830 | 1823767 | 102332755 | 56.08 | 0.9808 |
| 17 | 1822927 | 1983 | 0.00097 | 0.000970 | 1821936 | 100508987 | 55.14 | 0.9799 |
| 18 | 1820945 | 2501 | 0.00121 | 0.001210 | 1819694 | 98687051 | 54.20 | 0.9789 |
| 19 | 1818444 | 2796 | 0.00133 | 0.001320 | 1817046 | 96867357 | 53.27 | 0.9775 |
| 20 | 1815648 | 2856 | 0.00135 | 0.001350 | 1814220 | 95050311 | 52.35 | 0.9760 |
| 21 | 1812792 | 3071 | 0.00144 | 0.001440 | 1811257 | 93236090 | 51.43 | 0.9745 |
| 22 | 1809722 | 3128 | 0.00147 | 0.001470 | 1808158 | 91424833 | 50.52 | 0.9728 |
| 23 | 1806594 | 2957 | 0.00139 | 0.001390 | 1805115 | 89616676 | 49.61 | 0.9711 |
| 24 | 1803637 | 3093 | 0.00146 | 0.001450 | 1802090 | 87811560 | 48.69 | 0.9696 |
| 25 | 1800543 | 3251 | 0.00152 | 0.001520 | 1798918 | 86009470 | 47.77 | 0.9679 |
| 26 | 1797292 | 3059 | 0.00145 | 0.001450 | 1795763 | 84210552 | 46.85 | 0.9661 |
| 27 | 1794233 | 3004 | 0.00142 | 0.001410 | 1792731 | 82414790 | 45.93 | 0.9645 |
| 28 | 1791229 | 2932 | 0.00138 | 0.001380 | 1789763 | 80622059 | 45.01 | 0.9629 |
| 29 | 1788297 | 2946 | 0.00139 | 0.001390 | 1786824 | 78832295 | 44.08 | 0.9613 |
| 30 | 1785351 | 3038 | 0.00142 | 0.001420 | 1783832 | 77045471 | 43.15 | 0.9597 |
| 31 | 1782313 | 3211 | 0.00150 | 0.001490 | 1780708 | 75261639 | 42.23 | 0.9581 |
| 32 | 1779103 | 3208 | 0.00149 | 0.001490 | 1777499 | 73480931 | 41.30 | 0.9564 |
| 33 | 1775895 | 3438 | 0.00159 | 0.001590 | 1774176 | 71703432 | 40.38 | 0.9546 |
| 34 | 1772458 | 3955 | 0.00183 | 0.001820 | 1770480 | 69929256 | 39.45 | 0.9528 |
| 35 | 1768502 | 4465 | 0.00206 | 0.002050 | 1766270 | 68158776 | 38.54 | 0.9507 |
| 36 | 1764038 | 4804 | 0.00221 | 0.002210 | 1761636 | 66392506 | 37.64 | 0.9483 |
| 37 | 1759233 | 4881 | 0.00225 | 0.002240 | 1756793 | 64630870 | 36.74 | 0.9457 |
| 38 | 1754352 | 5278 | 0.00241 | 0.002410 | 1751713 | 62874078 | 35.84 | 0.9431 |
| 39 | 1749075 | 7052 | 0.00318 | 0.003170 | 1745548 | 61122364 | 34.95 | 0.9402 |
| 40 | 1742022 | 8393 | 0.00378 | 0.003780 | 1737825 | 59376816 | 34.08 | 0.9364 |
| Continued on next page | | | | |  | | | |

*Table 6 Continued from previous page*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 41 | 1733629 | 4856 | 0.00281 | 0.002800 | 1731201 | 57638990 | 33.25 | 0.9319 |
| 42 | 1728773 | 5151 | 0.00298 | 0.002980 | 1726197 | 55907790 | 32.34 | 0.9293 |
| 43 | 1723622 | 5466 | 0.00318 | 0.003170 | 1720889 | 54181592 | 31.43 | 0.9265 |
| 44 | 1718156 | 5798 | 0.00338 | 0.003370 | 1715257 | 52460703 | 30.53 | 0.9236 |
| 45 | 1712359 | 6230 | 0.00365 | 0.003640 | 1709244 | 50745446 | 29.63 | 0.9205 |
| 46 | 1706129 | 6734 | 0.00396 | 0.003950 | 1702761 | 49036202 | 28.74 | 0.9171 |
| 47 | 1699394 | 7221 | 0.00426 | 0.004250 | 1695784 | 47333441 | 27.85 | 0.9135 |
| 48 | 1692173 | 7823 | 0.00463 | 0.004620 | 1688262 | 45637657 | 26.97 | 0.9096 |
| 49 | 1684350 | 8250 | 0.00491 | 0.004900 | 1680225 | 43949395 | 26.09 | 0.9054 |
| 50 | 1676100 | 8947 | 0.00535 | 0.005340 | 1671627 | 42269170 | 25.22 | 0.9010 |
| 51 | 1667154 | 9628 | 0.00579 | 0.005780 | 1662340 | 40597543 | 24.35 | 0.8962 |
| 52 | 1657525 | 10372 | 0.00628 | 0.006260 | 1652340 | 38935204 | 23.49 | 0.8910 |
| 53 | 1647154 | 11055 | 0.00673 | 0.006710 | 1641626 | 37282864 | 22.63 | 0.8854 |
| 54 | 1636099 | 11780 | 0.00723 | 0.007200 | 1630209 | 35641238 | 21.78 | 0.8795 |
| 55 | 1624319 | 12782 | 0.00790 | 0.007870 | 1617928 | 34011029 | 20.94 | 0.8732 |
| 56 | 1611538 | 13579 | 0.00846 | 0.008430 | 1604748 | 32393100 | 20.10 | 0.8663 |
| 57 | 1597958 | 14442 | 0.00908 | 0.009040 | 1590737 | 30788352 | 19.27 | 0.8590 |
| 58 | 1583516 | 15546 | 0.00987 | 0.009820 | 1575743 | 29197615 | 18.44 | 0.8512 |
| 59 | 1567970 | 16326 | 0.01047 | 0.010410 | 1559807 | 27621872 | 17.62 | 0.8429 |
| 60 | 1551644 | 17525 | 0.01136 | 0.011290 | 1542881 | 26062065 | 16.80 | 0.8341 |
| 61 | 1534118 | 18483 | 0.01212 | 0.012050 | 1524877 | 24519184 | 15.98 | 0.8247 |
| 62 | 1515635 | 19773 | 0.01313 | 0.013050 | 1505749 | 22994307 | 15.17 | 0.8147 |
| 63 | 1495862 | 20765 | 0.01398 | 0.013880 | 1485480 | 21488558 | 14.37 | 0.8041 |
| 64 | 1475097 | 21758 | 0.01486 | 0.014750 | 1464218 | 20003079 | 13.56 | 0.7929 |
| 65 | 1453339 | 22877 | 0.01587 | 0.015740 | 1441901 | 18538861 | 12.76 | 0.7813 |
| 66 | 1430462 | 23980 | 0.01691 | 0.016760 | 1418472 | 17096960 | 11.95 | 0.7690 |
| 67 | 1406482 | 25068 | 0.01798 | 0.017820 | 1393948 | 15678487 | 11.15 | 0.7561 |
| 68 | 1381414 | 26563 | 0.01942 | 0.019230 | 1368133 | 14284539 | 10.34 | 0.7426 |
| 69 | 1354851 | 27959 | 0.02085 | 0.020640 | 1340872 | 12916406 | 9.53 | 0.7283 |
| 70 | 1326893 | 29609 | 0.02257 | 0.022310 | 1312088 | 11575534 | 8.72 | 0.7133 |
| 71 | 1297284 | 31106 | 0.02427 | 0.023980 | 1281731 | 10263446 | 7.91 | 0.6974 |
| 72 | 1266178 | 32765 | 0.02622 | 0.025880 | 1249795 | 8981715 | 7.09 | 0.6806 |
| 73 | 1233413 | 34450 | 0.02833 | 0.027930 | 1216188 | 7731920 | 6.27 | 0.6630 |
| 74 | 1198963 | 35999 | 0.03049 | 0.030030 | 1180963 | 6515732 | 5.43 | 0.6445 |
| 75 | 1162964 | 38106 | 0.03332 | 0.032770 | 1143911 | 5334769 | 4.59 | 0.6252 |
| 76 | 1124858 | 39950 | 0.03616 | 0.035520 | 1104883 | 4190858 | 3.73 | 0.6047 |
| 77 | 1084908 | 42055 | 0.03954 | 0.038760 | 1063880 | 3085975 | 2.84 | 0.5832 |
| 78 | 1042853 | 43755 | 0.04286 | 0.041960 | 1020975 | 2022095 | 1.94 | 0.5606 |
| 79 | 999097 | 45764 | 0.04689 | 0.045810 | 976215 | 1001121 | 1.00 | 0.5371 |
| 80 | 953333 | 49811 | 0.05366 | 0.052250 | 24906 | 24906 | 0.03 | 0.5125 |

Table 7: 2002 Female Life Table Generated using Mortality Rates of RH

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 0 | 1963747 | 12320 | 0.00635 | 0.006329 | 1957586.74 | 151482599.0 | 77.14 | 1.0000 |
| 1 | 1951426 | 802 | 0.00042 | 0.000415 | 1951025.425 | 149525012.2 | 76.62 | 0.9937 |
| 2 | 1950624 | 521 | 0.00027 | 0.000270 | 1950363.587 | 147573986.8 | 75.65 | 0.9933 |
| 3 | 1950102 | 354 | 0.00018 | 0.000183 | 1949925.589 | 145623623.2 | 74.67 | 0.9931 |
| 4 | 1949748 | 313 | 0.00016 | 0.000161 | 1949591.489 | 143673697.6 | 73.69 | 0.9929 |
| 5 | 1949434 | 249 | 0.00013 | 0.000127 | 1949310.056 | 141724106.1 | 72.70 | 0.9927 |
| 6 | 1949185 | 227 | 0.00012 | 0.000116 | 1949071.825 | 139774796.1 | 71.71 | 0.9926 |
| 7 | 1948958 | 187 | 9.50E-05 | 0.000095 | 1948864.365 | 137825724.3 | 70.72 | 0.9925 |
| 8 | 1948770 | 207 | 0.00011 | 0.000105 | 1948666.717 | 135876859.9 | 69.72 | 0.9924 |
| 9 | 1948562 | 194 | 9.80E-05 | 0.000098 | 1948465.501 | 133928193.2 | 68.73 | 0.9923 |
| 10 | 1948368 | 191 | 9.60E-05 | 0.000096 | 1948272.53 | 131979727.7 | 67.74 | 0.9922 |
| 11 | 1948176 | 197 | 9.90E-05 | 0.000099 | 1948078.004 | 130031455.1 | 66.75 | 0.9921 |
| 12 | 1947979 | 222 | 0.00011 | 0.000111 | 1947867.889 | 128083377.1 | 65.75 | 0.9920 |
| 13 | 1947756 | 291 | 0.00015 | 0.000145 | 1947610.916 | 126135509.3 | 64.76 | 0.9919 |
| 14 | 1947465 | 353 | 0.00018 | 0.000175 | 1947288.609 | 124187898.3 | 63.77 | 0.9917 |
| 15 | 1947112 | 423 | 0.00021 | 0.000209 | 1946900.412 | 122240609.7 | 62.78 | 0.9915 |
| 16 | 1946688 | 471 | 0.00023 | 0.000232 | 1946452.989 | 120293709.3 | 61.79 | 0.9913 |
| 17 | 1946217 | 533 | 0.00026 | 0.000260 | 1945950.222 | 118347256.3 | 60.81 | 0.9911 |
| 18 | 1945683 | 832 | 0.00040 | 0.000399 | 1945267.21 | 116401306.1 | 59.83 | 0.9908 |
| 19 | 1944851 | 960 | 0.00046 | 0.000458 | 1944371.003 | 114456038.9 | 58.85 | 0.9904 |
| 20 | 1943890 | 1152 | 0.00059 | 0.000593 | 1943314.624 | 112511667.9 | 57.88 | 0.9899 |
| 21 | 1942738 | 1244 | 0.00064 | 0.000641 | 1942115.983 | 110568353.3 | 56.91 | 0.9893 |
| 22 | 1941493 | 1323 | 0.00068 | 0.000682 | 1940831.711 | 108626237.3 | 55.95 | 0.9887 |
| 23 | 1940169 | 1404 | 0.00072 | 0.000724 | 1939467.8 | 106685405.6 | 54.99 | 0.9880 |
| 24 | 1938765 | 1457 | 0.00075 | 0.000752 | 1938037.011 | 104745937.8 | 54.03 | 0.9873 |
| 25 | 1937308 | 1489 | 0.00077 | 0.000769 | 1936563.7 | 102807900.8 | 53.07 | 0.9865 |
| 26 | 1935819 | 1503 | 0.00078 | 0.000777 | 1935067.318 | 100871337.1 | 52.11 | 0.9858 |
| 27 | 1934315 | 1531 | 0.00079 | 0.000792 | 1933549.859 | 98936269.7 | 51.15 | 0.9850 |
| 28 | 1932784 | 1510 | 0.00078 | 0.000782 | 1932028.75 | 97002719.9 | 50.19 | 0.9842 |
| 29 | 1931273 | 1554 | 0.00081 | 0.000805 | 1930496.302 | 95070691.1 | 49.23 | 0.9835 |
| 30 | 1929719 | 1564 | 0.00081 | 0.000811 | 1928937.093 | 93140194.8 | 48.27 | 0.9827 |
| 31 | 1928154 | 1530 | 0.00079 | 0.000794 | 1927389.736 | 91211257.7 | 47.30 | 0.9819 |
| 32 | 1926624 | 1561 | 0.00081 | 0.000811 | 1925843.632 | 89283868.0 | 46.34 | 0.9811 |
| 33 | 1925062 | 1595 | 0.00083 | 0.000829 | 1924265.095 | 87358024.4 | 45.38 | 0.9803 |
| 34 | 1923467 | 1640 | 0.00085 | 0.000853 | 1922647.478 | 85433759.3 | 44.42 | 0.9795 |
| 35 | 1921827 | 1682 | 0.00088 | 0.000876 | 1920986.077 | 83511111.8 | 43.45 | 0.9787 |
| 36 | 1920144 | 1702 | 0.00089 | 0.000887 | 1919293.479 | 81590125.7 | 42.49 | 0.9778 |
| 37 | 1918442 | 1737 | 0.00091 | 0.000906 | 1917573.611 | 79670832.2 | 41.53 | 0.9769 |
| 38 | 1916704 | 1699 | 0.00089 | 0.000887 | 1915855.269 | 77753258.6 | 40.57 | 0.9760 |
| 39 | 1915005 | 1766 | 0.00092 | 0.000923 | 1914122.22 | 75837403.4 | 39.60 | 0.9752 |
| 40 | 1913238 | 1747 | 0.00091 | 0.000914 | 1912364.902 | 73923281.1 | 38.64 | 0.9743 |
| Continued on next page | | | | |  | | | |

*Table 7: Continued from previous page*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 41 | 1911490 | 1757 | 0.00092 | 0.000920 | 1910612.07 | 72010916.2 | 37.67 | 0.9734 |
| 42 | 1909733 | 1750 | 0.00092 | 0.000917 | 1908857.977 | 70100304.2 | 36.71 | 0.9725 |
| 43 | 1907982 | 1790 | 0.00094 | 0.000939 | 1907087.388 | 68191446.2 | 35.74 | 0.9716 |
| 44 | 1906192 | 1853 | 0.00097 | 0.000973 | 1905265.099 | 66284358.8 | 34.77 | 0.9707 |
| 45 | 1904338 | 1905 | 0.00100 | 0.001000 | 1903385.543 | 64379093.7 | 33.81 | 0.9697 |
| 46 | 1902432 | 1988 | 0.00105 | 0.001045 | 1901438.447 | 62475708.2 | 32.84 | 0.9688 |
| 47 | 1900443 | 2055 | 0.00108 | 0.001081 | 1899416.41 | 60574269.7 | 31.87 | 0.9678 |
| 48 | 1898388 | 2117 | 0.00112 | 0.001115 | 1897330.116 | 58674853.3 | 30.91 | 0.9667 |
| 49 | 1896271 | 2283 | 0.00121 | 0.001204 | 1895129.59 | 56777523.2 | 29.94 | 0.9656 |
| 50 | 1893987 | 2377 | 0.00126 | 0.001255 | 1892799.097 | 54882393.6 | 28.98 | 0.9645 |
| 51 | 1891610 | 2491 | 0.00132 | 0.001317 | 1890364.67 | 52989594.5 | 28.01 | 0.9633 |
| 52 | 1889118 | 2575 | 0.00136 | 0.001363 | 1887831.419 | 51099229.8 | 27.05 | 0.9620 |
| 53 | 1886543 | 2686 | 0.00143 | 0.001424 | 1885200.712 | 49211398.4 | 26.09 | 0.9607 |
| 54 | 1883857 | 2806 | 0.00149 | 0.001490 | 1882454.138 | 47326197.7 | 25.12 | 0.9593 |
| 55 | 1881050 | 2920 | 0.00155 | 0.001553 | 1879590.327 | 45443743.6 | 24.16 | 0.9579 |
| 56 | 1878129 | 3038 | 0.00162 | 0.001618 | 1876610.77 | 43564153.2 | 23.20 | 0.9564 |
| 57 | 1875091 | 3229 | 0.00172 | 0.001723 | 1873476.717 | 41687542.5 | 22.23 | 0.9549 |
| 58 | 1871861 | 3289 | 0.00176 | 0.001757 | 1870216.925 | 39814065.8 | 21.27 | 0.9532 |
| 59 | 1868572 | 3597 | 0.00193 | 0.001925 | 1866773.434 | 37943848.8 | 20.31 | 0.9515 |
| 60 | 1864974 | 3750 | 0.00201 | 0.002011 | 1863099.589 | 36077075.4 | 19.34 | 0.9497 |
| 61 | 1861224 | 4027 | 0.00217 | 0.002164 | 1859210.856 | 34213975.8 | 18.38 | 0.9478 |
| 62 | 1857197 | 4148 | 0.00224 | 0.002234 | 1855123.304 | 32354765.0 | 17.42 | 0.9457 |
| 63 | 1853049 | 4523 | 0.00244 | 0.002441 | 1850787.616 | 30499641.6 | 16.46 | 0.9436 |
| 64 | 1848525 | 4912 | 0.00266 | 0.002657 | 1846069.76 | 28648854.0 | 15.50 | 0.9413 |
| 65 | 1843613 | 5147 | 0.00280 | 0.002792 | 1841039.794 | 26802784.3 | 14.54 | 0.9388 |
| 66 | 1838466 | 5613 | 0.00306 | 0.003053 | 1835659.301 | 24961744.5 | 13.58 | 0.9362 |
| 67 | 1832852 | 6027 | 0.00329 | 0.003289 | 1829838.838 | 23126085.2 | 12.62 | 0.9333 |
| 68 | 1826825 | 6464 | 0.00355 | 0.003539 | 1823592.782 | 21296246.3 | 11.66 | 0.9303 |
| 69 | 1820360 | 7096 | 0.00391 | 0.003898 | 1816812.237 | 19472653.6 | 10.70 | 0.9270 |
| 70 | 1813264 | 7520 | 0.00416 | 0.004147 | 1809503.864 | 17655841.3 | 9.74 | 0.9234 |
| 71 | 1805743 | 8373 | 0.00465 | 0.004637 | 1801556.909 | 15846337.5 | 8.78 | 0.9195 |
| 72 | 1797370 | 8799 | 0.00491 | 0.004896 | 1792970.159 | 14044780.5 | 7.81 | 0.9153 |
| 73 | 1788570 | 9559 | 0.00536 | 0.005345 | 1783790.563 | 12251810.4 | 6.85 | 0.9108 |
| 74 | 1779010 | 10550 | 0.00595 | 0.005930 | 1773735.833 | 10468019.8 | 5.88 | 0.9059 |
| 75 | 1768460 | 11167 | 0.00634 | 0.006315 | 1762876.865 | 8694284.0 | 4.92 | 0.9006 |
| 76 | 1757292 | 12177 | 0.00695 | 0.006930 | 1751204.059 | 6931407.1 | 3.94 | 0.8949 |
| 77 | 1745115 | 13415 | 0.00772 | 0.007687 | 1738407.535 | 5180203.1 | 2.97 | 0.8887 |
| 78 | 1731699 | 14602 | 0.00847 | 0.008432 | 1724398.861 | 3441795.5 | 1.99 | 0.8818 |
| 79 | 1717097. | 15716 | 0.00920 | 0.009153 | 1709239.626 | 1717396.7 | 1.00 | 0.8744 |
| 80 | 1701381 | 16314 | 0.00964 | 0.009589 | 8157.045477 | 8157.0 | 0.00 | 0.8664 |

Table 8: 2002 Male Life Table Generated using Mortality Rates of RH

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 0 | 20579797 | 15725 | 0.00775 | 0.007724 | 20571935 | 1514257075.0 | 73.58 | 1.0000 |
| 1 | 20564072 | 1061 | 0.00053 | 0.000526 | 20563542 | 1493685140.0 | 72.64 | 0.9992 |
| 2 | 20563012 | 667 | 0.00033 | 0.000331 | 20562678 | 1473121598.0 | 71.64 | 0.9992 |
| 3 | 20562344 | 519 | 0.00026 | 0.000257 | 20562085 | 1452558920.0 | 70.64 | 0.9992 |
| 4 | 20561825 | 395 | 0.00019 | 0.000194 | 20561628 | 1431996835.0 | 69.64 | 0.9991 |
| 5 | 20561430 | 346 | 0.00017 | 0.000169 | 20561257 | 1411435208.0 | 68.64 | 0.9991 |
| 6 | 20561084 | 270 | 0.00013 | 0.000132 | 20560949 | 1390873951.0 | 67.65 | 0.9991 |
| 7 | 20560815 | 261 | 1.27E-04 | 0.000127 | 20560684 | 1370313001.0 | 66.65 | 0.9991 |
| 8 | 20560554 | 258 | 0.00013 | 0.000125 | 20560425 | 1349752317.0 | 65.65 | 0.9991 |
| 9 | 20560296 | 242 | 1.17E-04 | 0.000117 | 20560175 | 1329191892.0 | 64.65 | 0.9991 |
| 10 | 20560054 | 249 | 1.20E-04 | 0.000120 | 20559929 | 1308631717.0 | 63.65 | 0.9990 |
| 11 | 20559805 | 258 | 1.24E-04 | 0.000124 | 20559676 | 1288071788.0 | 62.65 | 0.9990 |
| 12 | 20559547 | 313 | 0.00015 | 0.000150 | 20559390 | 1267512113.0 | 61.65 | 0.9990 |
| 13 | 20559234 | 393 | 0.00019 | 0.000188 | 20559037 | 1246952723.0 | 60.65 | 0.9990 |
| 14 | 20558840 | 510 | 0.00024 | 0.000243 | 20558585 | 1226393686.0 | 59.65 | 0.9990 |
| 15 | 20558330 | 797 | 0.00038 | 0.000378 | 20557931 | 1205835101.0 | 58.65 | 0.9990 |
| 16 | 20557533 | 1022 | 0.00048 | 0.000483 | 20557022 | 1185277170.0 | 57.66 | 0.9989 |
| 17 | 20556511 | 1351 | 0.00063 | 0.000631 | 20555835 | 1164720148.0 | 56.66 | 0.9989 |
| 18 | 20555160 | 2328 | 0.00107 | 0.001068 | 20553996 | 1144164313.0 | 55.66 | 0.9988 |
| 19 | 20552832 | 3031 | 0.00139 | 0.001384 | 20551316 | 1123610317.0 | 54.67 | 0.9987 |
| 20 | 20549801 | 27477 | 0.00134 | 0.001337 | 20536062 | 1103059001.0 | 53.68 | 0.9985 |
| 21 | 20522323 | 29695 | 0.00145 | 0.001447 | 20507476 | 1082522939.0 | 52.75 | 0.9972 |
| 22 | 20492629 | 29652 | 0.00145 | 0.001447 | 20477803 | 1062015463.0 | 51.82 | 0.9958 |
| 23 | 20462977 | 29384 | 0.00144 | 0.001436 | 20448285 | 1041537660.0 | 50.90 | 0.9943 |
| 24 | 20433593 | 29077 | 0.00142 | 0.001423 | 20419054 | 1021089375.0 | 49.97 | 0.9929 |
| 25 | 20404516 | 29056 | 0.00143 | 0.001424 | 20389988 | 1000670321.0 | 49.04 | 0.9915 |
| 26 | 20375460 | 29279 | 0.00144 | 0.001437 | 20360821 | 980280333.0 | 48.11 | 0.9901 |
| 27 | 20346181 | 29602 | 0.00146 | 0.001455 | 20331380 | 959919513.0 | 47.18 | 0.9887 |
| 28 | 20316579 | 30351 | 0.00150 | 0.001494 | 20301403 | 939588133.0 | 46.25 | 0.9872 |
| 29 | 20286228 | 30285 | 0.00149 | 0.001493 | 20271086 | 919286729.0 | 45.32 | 0.9857 |
| 30 | 20255943 | 30745 | 0.00152 | 0.001518 | 20240570 | 899015644.0 | 44.38 | 0.9843 |
| 31 | 20225198 | 31931 | 0.00158 | 0.001579 | 20209232 | 878775073.0 | 43.45 | 0.9828 |
| 32 | 20193267 | 32384 | 0.00161 | 0.001604 | 20177075 | 858565841.0 | 42.52 | 0.9812 |
| 33 | 20160883 | 33339 | 0.00166 | 0.001654 | 20144214 | 838388766.0 | 41.58 | 0.9796 |
| 34 | 20127544 | 33886 | 0.00169 | 0.001684 | 20110601 | 818244552.0 | 40.65 | 0.9780 |
| 35 | 20093658 | 34993 | 0.00174 | 0.001741 | 20076162 | 798133951.0 | 39.72 | 0.9764 |
| 36 | 20058665 | 36113 | 0.00180 | 0.001800 | 20040609 | 778057790.0 | 38.79 | 0.9747 |
| 37 | 20022552 | 37767 | 0.00189 | 0.001886 | 20003669 | 758017181.0 | 37.86 | 0.9729 |
| 38 | 19984785 | 39950 | 0.00200 | 0.001999 | 19964810 | 738013512.0 | 36.93 | 0.9711 |
| 39 | 19944836 | 40606 | 0.00204 | 0.002036 | 19924532 | 718048702.0 | 36.00 | 0.9692 |
| 40 | 19904229 | 43046 | 0.00217 | 0.002163 | 19882706 | 698124170.0 | 35.07 | 0.9672 |
| Continued on next page | | | | |  | | | |

*Table 8 Continued from previous page*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 41 | 19861183 | 46005 | 0.00232 | 0.002316 | 19838181 | 678241463.0 | 34.15 | 0.9651 |
| 42 | 19815179 | 48508 | 0.00245 | 0.002448 | 19790925 | 658403282.0 | 33.23 | 0.9629 |
| 43 | 19766671 | 51445 | 0.00261 | 0.002603 | 19740949 | 638612358.0 | 32.31 | 0.9605 |
| 44 | 19715226 | 54516 | 0.00277 | 0.002765 | 19687968 | 618871409.0 | 31.39 | 0.9580 |
| 45 | 19660710 | 58541 | 0.00298 | 0.002978 | 19631440 | 599183441.0 | 30.48 | 0.9553 |
| 46 | 19602169 | 63486 | 0.00324 | 0.003239 | 19570426 | 579552001.0 | 29.57 | 0.9525 |
| 47 | 19538683 | 68013 | 0.00349 | 0.003481 | 19504677 | 559981575.0 | 28.66 | 0.9494 |
| 48 | 19470670 | 73848 | 0.00380 | 0.003793 | 19433746 | 540476899.0 | 27.76 | 0.9461 |
| 49 | 19396822 | 77606 | 0.00401 | 0.004001 | 19358019 | 521043152.0 | 26.86 | 0.9425 |
| 50 | 19319216 | 84145 | 0.00437 | 0.004355 | 19277144 | 501685134.0 | 25.97 | 0.9388 |
| 51 | 19235071 | 90844 | 0.00473 | 0.004723 | 19189649 | 482407990.0 | 25.08 | 0.9347 |
| 52 | 19144228 | 97901 | 0.00513 | 0.005114 | 19095277 | 463218341.0 | 24.20 | 0.9302 |
| 53 | 19046326 | 104562 | 0.00551 | 0.005490 | 18994045 | 444123064.0 | 23.32 | 0.9255 |
| 54 | 18941764 | 111540 | 0.00591 | 0.005889 | 18885994 | 425129018.0 | 22.44 | 0.9204 |
| 55 | 18830224 | 121831 | 0.00649 | 0.006470 | 18769308 | 406243024.0 | 21.57 | 0.9150 |
| 56 | 18708393 | 129684 | 0.00696 | 0.006932 | 18643551 | 387473716.0 | 20.71 | 0.9091 |
| 57 | 18578709 | 138192 | 0.00747 | 0.007438 | 18509613 | 368830165.0 | 19.85 | 0.9028 |
| 58 | 18440517 | 149204 | 0.00812 | 0.008091 | 18365915 | 350320552.0 | 19.00 | 0.8961 |
| 59 | 18291313 | 156631 | 0.00860 | 0.008563 | 18212997 | 331954637.0 | 18.15 | 0.8888 |
| 60 | 18134682 | 168410 | 0.00933 | 0.009287 | 18050477 | 313741640.0 | 17.30 | 0.8812 |
| 61 | 17966272 | 178252 | 0.00997 | 0.009921 | 17877147 | 295691163.0 | 16.46 | 0.8730 |
| 62 | 17788021 | 190866 | 0.01079 | 0.010730 | 17692588 | 277814016.0 | 15.62 | 0.8643 |
| 63 | 17597155 | 201086 | 0.01149 | 0.011427 | 17496612 | 260121428.0 | 14.78 | 0.8551 |
| 64 | 17396069 | 211080 | 0.01221 | 0.012134 | 17290529 | 242624817.0 | 13.95 | 0.8453 |
| 65 | 17184988 | 221755 | 0.01299 | 0.012904 | 17074111 | 225334288.0 | 13.11 | 0.8350 |
| 66 | 16963233 | 233823 | 0.01388 | 0.013784 | 16846321 | 208260177.0 | 12.28 | 0.8243 |
| 67 | 16729410 | 244602 | 0.01473 | 0.014621 | 16607109 | 191413856.0 | 11.44 | 0.8129 |
| 68 | 16484808 | 260782 | 0.01595 | 0.015820 | 16354417 | 174806747.0 | 10.60 | 0.8010 |
| 69 | 16224026 | 276268 | 0.01718 | 0.017028 | 16085892 | 158452330.0 | 9.77 | 0.7884 |
| 70 | 15947758 | 293401 | 0.01857 | 0.018398 | 15801057 | 142366438.0 | 8.93 | 0.7749 |
| 71 | 15654356 | 310775 | 0.02005 | 0.019852 | 15498969 | 126565381.0 | 8.08 | 0.7607 |
| 72 | 15343581 | 327732 | 0.02159 | 0.021360 | 15179715 | 111066412.0 | 7.24 | 0.7456 |
| 73 | 15015849 | 347424 | 0.02341 | 0.023137 | 14842137 | 95886697.0 | 6.39 | 0.7296 |
| 74 | 14668425 | 364797 | 0.02518 | 0.024870 | 14486027 | 81044560.0 | 5.53 | 0.7128 |
| 75 | 14303628 | 389618 | 0.02762 | 0.027239 | 14108819 | 66558533.0 | 4.65 | 0.6950 |
| 76 | 13914010 | 412409 | 0.03009 | 0.029640 | 13707805 | 52449714.0 | 3.77 | 0.6761 |
| 77 | 13501601 | 439104 | 0.03306 | 0.032522 | 13282048 | 38741909.0 | 2.87 | 0.6561 |
| 78 | 13062496 | 461029 | 0.03593 | 0.035294 | 12831982 | 25459860.0 | 1.95 | 0.6347 |
| 79 | 12601467 | 487304 | 0.03944 | 0.038670 | 12357815 | 12627879.0 | 1.00 | 0.6123 |
| 80 | 12114163 | 540128 | 0.04561 | 0.044586 | 270064 | 270064.0 | 0.02 | 0.5886 |

Table 9: 1981 Female Life Table Generated using Mortality Rates of CBD

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 0 | 1768966 | 18857 | 0.01069 | 0.01063 | 1759537.54 | 112972441.35 | 63.86 | 1.0108 |
| 1 | 1750109 | 1383 | 0.00081 | 0.00081 | 1749417.51 | 111212903.82 | 63.55 | 1.0000 |
| 2 | 1748726 | 860 | 0.00050 | 0.00050 | 1748295.94 | 109463486.31 | 62.60 | 0.9992 |
| 3 | 1747866 | 614 | 0.00036 | 0.00036 | 1747558.96 | 107715190.37 | 61.63 | 0.9987 |
| 4 | 1747252 | 453 | 0.00026 | 0.00026 | 1747025.29 | 105967631.42 | 60.65 | 0.9984 |
| 5 | 1746799 | 389 | 0.00022 | 0.00022 | 1746604.20 | 104220606.12 | 59.66 | 0.9981 |
| 6 | 1746410 | 371 | 0.00021 | 0.00021 | 1746224.11 | 102474001.92 | 58.68 | 0.9979 |
| 7 | 1746038 | 346 | 0.00020 | 0.00020 | 1745865.56 | 100727777.81 | 57.69 | 0.9977 |
| 8 | 1745693 | 325 | 0.00019 | 0.00019 | 1745530.41 | 98981912.25 | 56.70 | 0.9975 |
| 9 | 1745368 | 301 | 0.00017 | 0.00017 | 1745217.65 | 97236381.84 | 55.71 | 0.9973 |
| 10 | 1745067 | 288 | 0.00016 | 0.00016 | 1744923.33 | 95491164.19 | 54.72 | 0.9971 |
| 11 | 1744779 | 274 | 0.00015 | 0.00015 | 1744642.53 | 93746240.86 | 53.73 | 0.9970 |
| 12 | 1744506 | 308 | 0.00017 | 0.00017 | 1744351.77 | 92001598.33 | 52.74 | 0.9968 |
| 13 | 1744198 | 358 | 0.00019 | 0.00019 | 1744019.01 | 90257246.56 | 51.75 | 0.9966 |
| 14 | 1743840 | 487 | 0.00026 | 0.00026 | 1743596.61 | 88513227.54 | 50.76 | 0.9964 |
| 15 | 1743353 | 599 | 0.00032 | 0.00032 | 1743053.50 | 86769630.94 | 49.77 | 0.9961 |
| 16 | 1742754 | 801 | 0.00042 | 0.00042 | 1742353.60 | 85026577.43 | 48.79 | 0.9958 |
| 17 | 1741953 | 825 | 0.00043 | 0.00043 | 1741541.04 | 83284223.83 | 47.81 | 0.9953 |
| 18 | 1741129 | 906 | 0.00046 | 0.00046 | 1740675.92 | 81542682.79 | 46.83 | 0.9949 |
| 19 | 1740223 | 899 | 0.00045 | 0.00045 | 1739773.42 | 79802006.88 | 45.86 | 0.9944 |
| 20 | 1739324 | 899 | 0.00044 | 0.00044 | 1738874.24 | 78062233.45 | 44.88 | 0.9938 |
| 21 | 1738425 | 1017 | 0.00050 | 0.00050 | 1737916.27 | 76323359.21 | 43.90 | 0.9933 |
| 22 | 1737408 | 1037 | 0.00051 | 0.00051 | 1736889.55 | 74585442.94 | 42.93 | 0.9927 |
| 23 | 1736371 | 948 | 0.00046 | 0.00046 | 1735897.46 | 72848553.39 | 41.95 | 0.9922 |
| 24 | 1735424 | 1036 | 0.00050 | 0.00050 | 1734905.88 | 71112655.92 | 40.98 | 0.9916 |
| 25 | 1734388 | 1093 | 0.00052 | 0.00052 | 1733841.61 | 69377750.05 | 40.00 | 0.9910 |
| 26 | 1733295 | 1171 | 0.00056 | 0.00056 | 1732709.64 | 67643908.44 | 39.03 | 0.9904 |
| 27 | 1732124 | 1149 | 0.00054 | 0.00054 | 1731549.75 | 65911198.79 | 38.05 | 0.9897 |
| 28 | 1730975 | 1294 | 0.00061 | 0.00061 | 1730328.10 | 64179649.05 | 37.08 | 0.9891 |
| 29 | 1729681 | 1320 | 0.00063 | 0.00063 | 1729020.86 | 62449320.95 | 36.10 | 0.9883 |
| 30 | 1728361 | 1416 | 0.00067 | 0.00067 | 1727652.63 | 60720300.08 | 35.13 | 0.9876 |
| 31 | 1726944 | 1484 | 0.00070 | 0.00070 | 1726202.24 | 58992647.46 | 34.16 | 0.9868 |
| 32 | 1725460 | 1614 | 0.00075 | 0.00075 | 1724653.05 | 57266445.22 | 33.19 | 0.9859 |
| 33 | 1723846 | 1779 | 0.00083 | 0.00083 | 1722956.49 | 55541792.17 | 32.22 | 0.9850 |
| 34 | 1722067 | 1995 | 0.00093 | 0.00092 | 1721069.33 | 53818835.68 | 31.25 | 0.9840 |
| 35 | 1720072 | 2316 | 0.00107 | 0.00107 | 1718913.68 | 52097766.35 | 30.29 | 0.9828 |
| 36 | 1717756 | 2463 | 0.00114 | 0.00113 | 1716524.39 | 50378852.66 | 29.33 | 0.9815 |
| 37 | 1715293 | 2585 | 0.00119 | 0.00119 | 1714000.59 | 48662328.27 | 28.37 | 0.9801 |
| 38 | 1712708 | 2896 | 0.00133 | 0.00133 | 1711260.06 | 46948327.69 | 27.41 | 0.9786 |
| 39 | 1709812 | 3552 | 0.00162 | 0.00162 | 1708036.03 | 45237067.63 | 26.46 | 0.9770 |
| 40 | 1706260 | 4475 | 0.00205 | 0.00204 | 1704022.64 | 43529031.60 | 25.51 | 0.9749 |
| Continued on next page | | | | |  | | | |

*Table 9 Continued from previous page*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 41 | 1701785 | 2756 | 0.00162 | 0.00162 | 1700407.10 | 41825008.95 | 24.58 | 0.9724 |
| 42 | 1699029 | 3137 | 0.00185 | 0.00185 | 1697460.47 | 40124601.86 | 23.62 | 0.9708 |
| 43 | 1695892 | 3585 | 0.00212 | 0.00211 | 1694099.66 | 38427141.39 | 22.66 | 0.9690 |
| 44 | 1692307 | 4112 | 0.00243 | 0.00243 | 1690251.11 | 36733041.74 | 21.71 | 0.9670 |
| 45 | 1688195 | 4737 | 0.00281 | 0.00281 | 1685826.34 | 35042790.63 | 20.76 | 0.9646 |
| 46 | 1683458 | 5477 | 0.00326 | 0.00325 | 1680719.02 | 33356964.29 | 19.81 | 0.9619 |
| 47 | 1677980 | 6358 | 0.00380 | 0.00379 | 1674801.52 | 31676245.27 | 18.88 | 0.9588 |
| 48 | 1671623 | 7406 | 0.00444 | 0.00443 | 1667919.98 | 30001443.75 | 17.95 | 0.9552 |
| 49 | 1664217 | 8658 | 0.00522 | 0.00520 | 1659888.22 | 28333523.77 | 17.03 | 0.9509 |
| 50 | 1655559 | 10155 | 0.00615 | 0.00613 | 1650481.55 | 26673635.55 | 16.11 | 0.9460 |
| 51 | 1645404 | 11948 | 0.00729 | 0.00726 | 1639429.81 | 25023153.99 | 15.21 | 0.9402 |
| 52 | 1633456 | 14099 | 0.00867 | 0.00863 | 1626406.14 | 23383724.19 | 14.32 | 0.9333 |
| 53 | 1619357 | 16679 | 0.01035 | 0.01030 | 1611017.17 | 21757318.05 | 13.44 | 0.9253 |
| 54 | 1602678 | 19771 | 0.01241 | 0.01234 | 1592792.27 | 20146300.88 | 12.57 | 0.9158 |
| 55 | 1582907 | 23474 | 0.01494 | 0.01483 | 1571169.53 | 18553508.61 | 11.72 | 0.9045 |
| 56 | 1559432 | 27898 | 0.01805 | 0.01789 | 1545483.18 | 16982339.07 | 10.89 | 0.8910 |
| 57 | 1531534 | 33161 | 0.02189 | 0.02165 | 1514953.52 | 15436855.90 | 10.08 | 0.8751 |
| 58 | 1498373 | 39381 | 0.02663 | 0.02628 | 1478682.56 | 13921902.38 | 9.29 | 0.8562 |
| 59 | 1458992 | 46668 | 0.03251 | 0.03199 | 1435658.26 | 12443219.81 | 8.53 | 0.8337 |
| 60 | 1412324 | 55098 | 0.03979 | 0.03901 | 1384775.17 | 11007561.55 | 7.79 | 0.8070 |
| 61 | 1357226 | 64683 | 0.04883 | 0.04766 | 1324884.64 | 9622786.38 | 7.09 | 0.7755 |
| 62 | 1292543 | 75318 | 0.06004 | 0.05827 | 1254884.29 | 8297901.74 | 6.42 | 0.7386 |
| 63 | 1217225 | 86728 | 0.07392 | 0.07125 | 1173861.40 | 7043017.46 | 5.79 | 0.6955 |
| 64 | 1130498 | 98388 | 0.09105 | 0.08703 | 1081303.59 | 5869156.06 | 5.19 | 0.6460 |
| 65 | 1032110 | 109465 | 0.11212 | 0.10606 | 977377.11 | 4787852.47 | 4.64 | 0.5897 |
| 66 | 922645 | 118797 | 0.13783 | 0.12876 | 863246.29 | 3810475.36 | 4.13 | 0.5272 |
| 67 | 803848 | 124953 | 0.16894 | 0.15544 | 741371.70 | 2947229.07 | 3.67 | 0.4593 |
| 68 | 678895 | 126451 | 0.20611 | 0.18626 | 615670.02 | 2205857.36 | 3.25 | 0.3879 |
| 69 | 552445 | 122127 | 0.24983 | 0.22107 | 491381.23 | 1590187.34 | 2.88 | 0.3157 |
| 70 | 430318 | 111606 | 0.30024 | 0.25936 | 374514.97 | 1098806.11 | 2.55 | 0.2459 |
| 71 | 318712 | 95683 | 0.35699 | 0.30022 | 270870.58 | 724291.14 | 2.27 | 0.1821 |
| 72 | 223029 | 76359 | 0.41912 | 0.34237 | 184849.61 | 453420.56 | 2.03 | 0.1274 |
| 73 | 146670 | 56367 | 0.48501 | 0.38431 | 118486.82 | 268570.95 | 1.83 | 0.0838 |
| 74 | 90303 | 38333 | 0.55251 | 0.42450 | 71136.79 | 150084.14 | 1.66 | 0.0516 |
| 75 | 51970 | 23989 | 0.61915 | 0.46160 | 39975.39 | 78947.35 | 1.52 | 0.0297 |
| 76 | 27981 | 13841 | 0.68254 | 0.49467 | 21060.14 | 38971.96 | 1.39 | 0.0160 |
| 77 | 14140 | 7398 | 0.74065 | 0.52319 | 10440.70 | 17911.82 | 1.27 | 0.0081 |
| 78 | 6742 | 3688 | 0.79209 | 0.54710 | 4897.60 | 7471.12 | 1.11 | 0.0039 |
| 79 | 3053 | 1730 | 0.83619 | 0.56664 | 2188.29 | 2573.52 | 0.84 | 0.0017 |
| 80 | 1323 | 770 | 0.87293 | 0.58227 | 385.23 | 385.23 | 0.29 | 0.0008 |

Table 10: 1981 Male Life Table Generated using Mortality Rates of CBD

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 0 | 1860272 | 24462 | 0.01324 | 0.01315 | 1848041.00 | 113238185.00 | 60.87 | 1.0000 |
| 1 | 1835810 | 1741 | 0.00098 | 0.00097 | 1834939.00 | 111390144.00 | 60.68 | 0.9869 |
| 2 | 1834069 | 1171 | 0.00066 | 0.00066 | 1833484.00 | 109555204.00 | 59.73 | 0.9859 |
| 3 | 1832899 | 784 | 0.00044 | 0.00043 | 1832506.00 | 107721721.00 | 58.77 | 0.9853 |
| 4 | 1832114 | 719 | 0.00039 | 0.00039 | 1831755.00 | 105889214.00 | 57.80 | 0.9849 |
| 5 | 1831395 | 564 | 0.00031 | 0.00031 | 1831113.00 | 104057459.00 | 56.82 | 0.9845 |
| 6 | 1830831 | 584 | 0.00032 | 0.00032 | 1830539.00 | 102226346.00 | 55.84 | 0.9842 |
| 7 | 1830247 | 476 | 0.00026 | 0.00026 | 1830009.00 | 100395807.00 | 54.85 | 0.9839 |
| 8 | 1829771 | 498 | 0.00028 | 0.00028 | 1829522.00 | 98565798.00 | 53.87 | 0.9836 |
| 9 | 1829273 | 422 | 0.00022 | 0.00022 | 1829062.00 | 96736276.00 | 52.88 | 0.9833 |
| 10 | 1828851 | 451 | 0.00024 | 0.00023 | 1828626.00 | 94907214.00 | 51.89 | 0.9831 |
| 11 | 1828401 | 432 | 0.00023 | 0.00023 | 1828184.00 | 93078588.00 | 50.91 | 0.9829 |
| 12 | 1827968 | 520 | 0.00027 | 0.00027 | 1827708.00 | 91250404.00 | 49.92 | 0.9826 |
| 13 | 1827448 | 673 | 0.00035 | 0.00035 | 1827111.00 | 89422695.00 | 48.93 | 0.9824 |
| 14 | 1826775 | 980 | 0.00050 | 0.00050 | 1826285.00 | 87595584.00 | 47.95 | 0.9820 |
| 15 | 1825794 | 1187 | 0.00059 | 0.00059 | 1825201.00 | 85769300.00 | 46.98 | 0.9815 |
| 16 | 1824607 | 1680 | 0.00083 | 0.00083 | 1823767.00 | 83944099.00 | 46.01 | 0.9808 |
| 17 | 1822927 | 1983 | 0.00097 | 0.00097 | 1821936.00 | 82120332.00 | 45.05 | 0.9799 |
| 18 | 1820945 | 2501 | 0.00121 | 0.00121 | 1819694.00 | 80298396.00 | 44.10 | 0.9789 |
| 19 | 1818444 | 2796 | 0.00133 | 0.00132 | 1817046.00 | 78478701.00 | 43.16 | 0.9775 |
| 20 | 1815648 | 2856 | 0.00135 | 0.00135 | 1814220.00 | 76661655.00 | 42.22 | 0.9760 |
| 21 | 1812792 | 3071 | 0.00144 | 0.00144 | 1811257.00 | 74847434.00 | 41.29 | 0.9745 |
| 22 | 1809722 | 3128 | 0.00147 | 0.00147 | 1808158.00 | 73036177.00 | 40.36 | 0.9728 |
| 23 | 1806594 | 2957 | 0.00139 | 0.00139 | 1805115.00 | 71228020.00 | 39.43 | 0.9711 |
| 24 | 1803637 | 3093 | 0.00146 | 0.00145 | 1802090.00 | 69422904.00 | 38.49 | 0.9696 |
| 25 | 1800543 | 3251 | 0.00152 | 0.00152 | 1798918.00 | 67620814.00 | 37.56 | 0.9679 |
| 26 | 1797292 | 3059 | 0.00145 | 0.00145 | 1795763.00 | 65821897.00 | 36.62 | 0.9661 |
| 27 | 1794233 | 3004 | 0.00142 | 0.00141 | 1792731.00 | 64026134.00 | 35.68 | 0.9645 |
| 28 | 1791229 | 2932 | 0.00138 | 0.00138 | 1789763.00 | 62233403.00 | 34.74 | 0.9629 |
| 29 | 1788297 | 2946 | 0.00139 | 0.00139 | 1786824.00 | 60443639.00 | 33.80 | 0.9613 |
| 30 | 1785351 | 3038 | 0.00142 | 0.00142 | 1783832.00 | 58656815.00 | 32.85 | 0.9597 |
| 31 | 1782313 | 3211 | 0.00150 | 0.00149 | 1780708.00 | 56872983.00 | 31.91 | 0.9581 |
| 32 | 1779103 | 3208 | 0.00149 | 0.00149 | 1777499.00 | 55092275.00 | 30.97 | 0.9564 |
| 33 | 1775895 | 3438 | 0.00159 | 0.00159 | 1774176.00 | 53314776.00 | 30.02 | 0.9546 |
| 34 | 1772458 | 3955 | 0.00183 | 0.00182 | 1770480.00 | 51540600.00 | 29.08 | 0.9528 |
| 35 | 1768502 | 4465 | 0.00206 | 0.00205 | 1766270.00 | 49770120.00 | 28.14 | 0.9507 |
| 36 | 1764038 | 4804 | 0.00221 | 0.00221 | 1761636.00 | 48003850.00 | 27.21 | 0.9483 |
| 37 | 1759233 | 4881 | 0.00225 | 0.00224 | 1756793.00 | 46242214.00 | 26.29 | 0.9457 |
| 38 | 1754352 | 5278 | 0.00241 | 0.00241 | 1751713.00 | 44485422.00 | 25.36 | 0.9431 |
| 39 | 1749075 | 7052 | 0.00318 | 0.00317 | 1745548.00 | 42733708.00 | 24.43 | 0.9402 |
| 40 | 1742022 | 8393 | 0.00378 | 0.00378 | 1737825.00 | 40988160.00 | 23.53 | 0.9364 |
| Continued on next page | | | | |  | | | |

*Table 10 Continued from previous page*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 41 | 1733629 | 6057 | 0.00350 | 0.00349 | 1730600.00 | 39250335.00 | 22.64 | 0.9319 |
| 42 | 1727572 | 6779 | 0.00393 | 0.00392 | 1724182.00 | 37519734.00 | 21.72 | 0.9287 |
| 43 | 1720792 | 7613 | 0.00443 | 0.00442 | 1716986.00 | 35795552.00 | 20.80 | 0.9250 |
| 44 | 1713179 | 8573 | 0.00502 | 0.00500 | 1708892.00 | 34078567.00 | 19.89 | 0.9209 |
| 45 | 1704606 | 9685 | 0.00570 | 0.00568 | 1699763.00 | 32369674.00 | 18.99 | 0.9163 |
| 46 | 1694920 | 10969 | 0.00649 | 0.00647 | 1689436.00 | 30669911.00 | 18.10 | 0.9111 |
| 47 | 1683951 | 12457 | 0.00743 | 0.00740 | 1677722.00 | 28980476.00 | 17.21 | 0.9052 |
| 48 | 1671494 | 14181 | 0.00852 | 0.00848 | 1664404.00 | 27302753.00 | 16.33 | 0.8985 |
| 49 | 1657313 | 16180 | 0.00981 | 0.00976 | 1649223.00 | 25638350.00 | 15.47 | 0.8909 |
| 50 | 1641133 | 18496 | 0.01133 | 0.01127 | 1631885.00 | 23989126.00 | 14.62 | 0.8822 |
| 51 | 1622637 | 21180 | 0.01314 | 0.01305 | 1612047.00 | 22357241.00 | 13.78 | 0.8723 |
| 52 | 1601457 | 24284 | 0.01528 | 0.01516 | 1589315.00 | 20745194.00 | 12.95 | 0.8609 |
| 53 | 1577173 | 27869 | 0.01783 | 0.01767 | 1563238.00 | 19155880.00 | 12.15 | 0.8478 |
| 54 | 1549304 | 31991 | 0.02087 | 0.02065 | 1533308.00 | 17592641.00 | 11.36 | 0.8328 |
| 55 | 1517313 | 36712 | 0.02449 | 0.02420 | 1498957.00 | 16059333.00 | 10.58 | 0.8156 |
| 56 | 1480601 | 42082 | 0.02883 | 0.02842 | 1459560.00 | 14560376.00 | 9.83 | 0.7959 |
| 57 | 1438519 | 48139 | 0.03404 | 0.03346 | 1414449.00 | 13100817.00 | 9.11 | 0.7733 |
| 58 | 1390380 | 54892 | 0.04028 | 0.03948 | 1362934.00 | 11686368.00 | 8.41 | 0.7474 |
| 59 | 1335488 | 62304 | 0.04778 | 0.04665 | 1304336.00 | 10323434.00 | 7.73 | 0.7179 |
| 60 | 1273184 | 70275 | 0.05678 | 0.05520 | 1238046.00 | 9019098.00 | 7.08 | 0.6844 |
| 61 | 1202909 | 78610 | 0.06758 | 0.06535 | 1163604.00 | 7781051.00 | 6.47 | 0.6466 |
| 62 | 1124299 | 86998 | 0.08054 | 0.07738 | 1080800.00 | 6617447.00 | 5.89 | 0.6044 |
| 63 | 1037301 | 94983 | 0.09604 | 0.09157 | 989809.00 | 5536648.00 | 5.34 | 0.5576 |
| 64 | 942317 | 101956 | 0.11451 | 0.10820 | 891339.00 | 4546839.00 | 4.83 | 0.5065 |
| 65 | 840361 | 107173 | 0.13643 | 0.12753 | 786775.00 | 3655499.00 | 4.35 | 0.4517 |
| 66 | 733188 | 109823 | 0.16227 | 0.14979 | 678276.00 | 2868725.00 | 3.91 | 0.3941 |
| 67 | 623365 | 109144 | 0.19248 | 0.17509 | 568793.00 | 2190448.00 | 3.51 | 0.3351 |
| 68 | 514220 | 104602 | 0.22743 | 0.20342 | 461919.00 | 1621656.00 | 3.15 | 0.2764 |
| 69 | 409618 | 96088 | 0.26733 | 0.23458 | 361574.00 | 1159736.00 | 2.83 | 0.2202 |
| 70 | 313530 | 84076 | 0.31219 | 0.26816 | 271493.00 | 798162.00 | 2.55 | 0.1685 |
| 71 | 229455 | 69642 | 0.36171 | 0.30351 | 194634.00 | 526669.00 | 2.30 | 0.1233 |
| 72 | 159812 | 54305 | 0.41522 | 0.33981 | 132660.00 | 332036.00 | 2.08 | 0.0859 |
| 73 | 105507 | 39678 | 0.47172 | 0.37607 | 85668.00 | 199376.00 | 1.89 | 0.0567 |
| 74 | 65829 | 27076 | 0.52985 | 0.41131 | 52291.00 | 113708.00 | 1.73 | 0.0354 |
| 75 | 38753 | 17229 | 0.58806 | 0.44460 | 30138.00 | 61416.00 | 1.58 | 0.0208 |
| 76 | 21524 | 10228 | 0.64473 | 0.47520 | 16410.00 | 31278.00 | 1.45 | 0.0116 |
| 77 | 11296 | 5677 | 0.69838 | 0.50261 | 8457.00 | 14868.00 | 1.32 | 0.0061 |
| 78 | 5618 | 2959 | 0.74779 | 0.52659 | 4139.00 | 6411.00 | 1.14 | 0.0030 |
| 79 | 2660 | 1455 | 0.79212 | 0.54712 | 1932.00 | 2272.00 | 0.85 | 0.0014 |
| 80 | 1205 | 680 | 0.83093 | 0.56436 | 340 | 340 | 0.28 | 0.0006 |

Table 11: 2002 Female Life Table Generated using Mortality Rates of CBD

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 0 | 1963747 | 12321 | 0.00635 | 0.00633 | 1957586.74 | 105648894.32 | 53.80 | 1.0000 |
| 1 | 1951426 | 802 | 0.00042 | 0.00042 | 1951025.43 | 103691307.58 | 53.14 | 0.9937 |
| 2 | 1950624 | 522 | 0.00027 | 0.00027 | 1950363.59 | 101740282.15 | 52.16 | 0.9933 |
| 3 | 1950103 | 354 | 0.00018 | 0.00018 | 1949925.59 | 99789918.57 | 51.17 | 0.9931 |
| 4 | 1949748 | 314 | 0.00016 | 0.00016 | 1949591.49 | 97839992.98 | 50.18 | 0.9929 |
| 5 | 1949435 | 249 | 0.00013 | 0.00013 | 1949310.06 | 95890401.49 | 49.19 | 0.9927 |
| 6 | 1949186 | 227 | 0.00012 | 0.00012 | 1949071.83 | 93941091.43 | 48.20 | 0.9926 |
| 7 | 1948958 | 188 | 0.00010 | 0.00010 | 1948864.36 | 91992019.61 | 47.20 | 0.9925 |
| 8 | 1948771 | 208 | 0.00011 | 0.00011 | 1948666.72 | 90043155.24 | 46.21 | 0.9924 |
| 9 | 1948563 | 195 | 0.00010 | 0.00010 | 1948465.50 | 88094488.53 | 45.21 | 0.9923 |
| 10 | 1948368 | 191 | 0.00010 | 0.00010 | 1948272.53 | 86146023.03 | 44.21 | 0.9922 |
| 11 | 1948177 | 198 | 0.00010 | 0.00010 | 1948078.00 | 84197750.50 | 43.22 | 0.9921 |
| 12 | 1947979 | 222 | 0.00011 | 0.00011 | 1947867.89 | 82249672.49 | 42.22 | 0.9920 |
| 13 | 1947757 | 292 | 0.00015 | 0.00015 | 1947610.92 | 80301804.60 | 41.23 | 0.9919 |
| 14 | 1947465 | 353 | 0.00018 | 0.00018 | 1947288.61 | 78354193.69 | 40.23 | 0.9917 |
| 15 | 1947112 | 423 | 0.00021 | 0.00021 | 1946900.41 | 76406905.08 | 39.24 | 0.9915 |
| 16 | 1946689 | 472 | 0.00023 | 0.00023 | 1946452.99 | 74460004.67 | 38.25 | 0.9913 |
| 17 | 1946217 | 534 | 0.00026 | 0.00026 | 1945950.22 | 72513551.68 | 37.26 | 0.9911 |
| 18 | 1945683 | 832 | 0.00040 | 0.00040 | 1945267.21 | 70567601.45 | 36.27 | 0.9908 |
| 19 | 1944851 | 960 | 0.00046 | 0.00046 | 1944371.00 | 68622334.24 | 35.28 | 0.9904 |
| 20 | 1943891 | 1149 | 0.00059 | 0.00059 | 1943316.57 | 66677963.24 | 34.30 | 0.9899 |
| 21 | 1942742 | 1251 | 0.00064 | 0.00064 | 1942116.95 | 64734646.67 | 33.32 | 0.9893 |
| 22 | 1941492 | 1366 | 0.00070 | 0.00070 | 1940808.43 | 62792529.72 | 32.34 | 0.9887 |
| 23 | 1940125 | 1501 | 0.00077 | 0.00077 | 1939374.73 | 60851721.29 | 31.36 | 0.9880 |
| 24 | 1938624 | 1655 | 0.00085 | 0.00085 | 1937796.75 | 58912346.56 | 30.39 | 0.9872 |
| 25 | 1936969 | 1832 | 0.00095 | 0.00095 | 1936053.56 | 56974549.82 | 29.41 | 0.9864 |
| 26 | 1935138 | 2037 | 0.00105 | 0.00105 | 1934119.49 | 55038496.26 | 28.44 | 0.9854 |
| 27 | 1933101 | 2276 | 0.00118 | 0.00118 | 1931963.25 | 53104376.77 | 27.47 | 0.9844 |
| 28 | 1930825 | 2551 | 0.00132 | 0.00132 | 1929549.89 | 51172413.52 | 26.50 | 0.9832 |
| 29 | 1928274 | 2873 | 0.00149 | 0.00149 | 1926838.00 | 49242863.63 | 25.54 | 0.9819 |
| 30 | 1925402 | 3249 | 0.00169 | 0.00169 | 1923776.91 | 47316025.63 | 24.57 | 0.9805 |
| 31 | 1922152 | 3689 | 0.00192 | 0.00192 | 1920307.83 | 45392248.71 | 23.62 | 0.9788 |
| 32 | 1918463 | 4206 | 0.00220 | 0.00219 | 1916360.17 | 43471940.88 | 22.66 | 0.9769 |
| 33 | 1914257 | 4816 | 0.00252 | 0.00252 | 1911848.99 | 41555580.71 | 21.71 | 0.9748 |
| 34 | 1909441 | 5533 | 0.00290 | 0.00290 | 1906674.44 | 39643731.72 | 20.76 | 0.9723 |
| 35 | 1903908 | 6385 | 0.00336 | 0.00335 | 1900715.61 | 37737057.28 | 19.82 | 0.9695 |
| 36 | 1897523 | 7393 | 0.00390 | 0.00390 | 1893826.61 | 35836341.67 | 18.89 | 0.9663 |
| 37 | 1890130 | 8592 | 0.00456 | 0.00455 | 1885833.95 | 33942515.06 | 17.96 | 0.9625 |
| 38 | 1881538 | 10021 | 0.00534 | 0.00533 | 1876527.71 | 32056681.11 | 17.04 | 0.9581 |
| 39 | 1871517 | 11727 | 0.00629 | 0.00627 | 1865653.66 | 30180153.41 | 16.13 | 0.9530 |
| 40 | 1859790 | 13767 | 0.00743 | 0.00740 | 1852906.41 | 28314499.75 | 15.22 | 0.9471 |
| Continued on next page | | | | |  | | | |

*Table 11 Continued from previous page*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 41 | 1846023 | 16207 | 0.00882 | 0.00878 | 1837919.56 | 26461593.33 | 14.33 | 0.9401 |
| 42 | 1829816 | 19127 | 0.01051 | 0.01045 | 1820252.71 | 24623673.77 | 13.46 | 0.9318 |
| 43 | 1810689 | 22620 | 0.01257 | 0.01249 | 1799379.34 | 22803421.06 | 12.59 | 0.9221 |
| 44 | 1788069 | 26795 | 0.01510 | 0.01499 | 1774671.86 | 21004041.72 | 11.75 | 0.9105 |
| 45 | 1761274 | 31770 | 0.01820 | 0.01804 | 1745389.01 | 19229369.86 | 10.92 | 0.8969 |
| 46 | 1729504 | 37676 | 0.02203 | 0.02178 | 1710665.84 | 17483980.86 | 10.11 | 0.8807 |
| 47 | 1691828 | 44642 | 0.02674 | 0.02639 | 1669507.08 | 15773315.01 | 9.32 | 0.8615 |
| 48 | 1647186 | 52785 | 0.03257 | 0.03205 | 1620793.97 | 14103807.93 | 8.56 | 0.8388 |
| 49 | 1594402 | 62186 | 0.03978 | 0.03900 | 1563308.44 | 12483013.96 | 7.83 | 0.8119 |
| 50 | 1532215 | 72856 | 0.04872 | 0.04755 | 1495787.28 | 10919705.52 | 7.13 | 0.7803 |
| 51 | 1459359 | 84680 | 0.05978 | 0.05803 | 1417019.38 | 9423918.23 | 6.46 | 0.7432 |
| 52 | 1374679 | 97352 | 0.07345 | 0.07082 | 1326003.15 | 8006898.86 | 5.82 | 0.7000 |
| 53 | 1277327 | 110302 | 0.09031 | 0.08635 | 1222175.82 | 6680895.70 | 5.23 | 0.6505 |
| 54 | 1167025 | 122618 | 0.11101 | 0.10507 | 1105715.49 | 5458719.89 | 4.68 | 0.5943 |
| 55 | 1044406 | 133036 | 0.13626 | 0.12738 | 977888.06 | 4353004.40 | 4.17 | 0.5318 |
| 56 | 911370 | 139996 | 0.16678 | 0.15361 | 841372.12 | 3375116.34 | 3.70 | 0.4641 |
| 57 | 771374 | 141864 | 0.20323 | 0.18391 | 700442.21 | 2533744.22 | 3.28 | 0.3928 |
| 58 | 629510 | 137342 | 0.24612 | 0.21817 | 560839.20 | 1833302.01 | 2.91 | 0.3206 |
| 59 | 492168 | 125961 | 0.29562 | 0.25593 | 429188.05 | 1272462.81 | 2.59 | 0.2506 |
| 60 | 366208 | 108517 | 0.35144 | 0.29633 | 311949.33 | 843274.76 | 2.30 | 0.1865 |
| 61 | 257691 | 87135 | 0.41270 | 0.33814 | 214123.55 | 531325.43 | 2.06 | 0.1312 |
| 62 | 170556 | 64794 | 0.47787 | 0.37990 | 138159.39 | 317201.88 | 1.86 | 0.0869 |
| 63 | 105763 | 44430 | 0.54488 | 0.42009 | 83547.63 | 179042.49 | 1.69 | 0.0539 |
| 64 | 61333 | 28052 | 0.61135 | 0.45738 | 47306.46 | 95494.86 | 1.56 | 0.0312 |
| 65 | 33280 | 16333 | 0.67487 | 0.49078 | 25113.61 | 48188.40 | 1.45 | 0.0169 |
| 66 | 16947 | 8808 | 0.73341 | 0.51973 | 12543.04 | 23074.79 | 1.36 | 0.0086 |
| 67 | 8139 | 4429 | 0.78551 | 0.54411 | 5924.81 | 10531.75 | 1.29 | 0.0041 |
| 68 | 3711 | 2093 | 0.83040 | 0.56412 | 2663.92 | 4606.94 | 1.24 | 0.0019 |
| 69 | 1617 | 938 | 0.86798 | 0.58020 | 1148.14 | 1943.02 | 1.20 | 0.0008 |
| 70 | 679 | 403 | 0.89865 | 0.59288 | 477.68 | 794.88 | 1.17 | 0.0003 |
| 71 | 276 | 167 | 0.92314 | 0.60273 | 193.11 | 317.20 | 1.15 | 0.0001 |
| 72 | 110 | 67 | 0.94233 | 0.61028 | 76.30 | 124.09 | 1.13 | 0.0001 |
| 73 | 43 | 26 | 0.95714 | 0.61601 | 29.61 | 47.78 | 1.12 | 0.0000 |
| 74 | 16 | 10 | 0.96840 | 0.62031 | 11.34 | 18.17 | 1.11 | 0.0000 |
| 75 | 6 | 4 | 0.97687 | 0.62351 | 4.29 | 6.83 | 1.10 | 0.0000 |
| 76 | 2 | 1 | 0.98319 | 0.62588 | 1.61 | 2.54 | 1.08 | 0.0000 |
| 77 | 1 | 1 | 0.98785 | 0.62762 | 0.60 | 0.92 | 1.05 | 0.0000 |
| 78 | 0 | 0 | 0.99127 | 0.62890 | 0.22 | 0.32 | 0.98 | 0.0000 |
| 79 | 0.12 | 0.08 | 0.99376 | 0.629818 | 0.08 | 0.1 | 0.8 | 0 |
| 80 | 0.04 | 0.03 | 0.99556 | 0.630485 | 0.0141722486 | 0.01 | 0.32 | 0 |

Table 12: 2002 Male Life Table Generated using Mortality Rates of CBD

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 0 | 20579797 | 15725 | 0.00775 | 0.00772 | 20571934.73 | 1097897674.00 | 53.35 | 1.0000 |
| 1 | 20564072 | 1061 | 0.00053 | 0.00053 | 20563542.02 | 1077325739.00 | 52.39 | 0.9992 |
| 2 | 20563012 | 667 | 0.00033 | 0.00033 | 20562677.92 | 1056762197.00 | 51.39 | 0.9992 |
| 3 | 20562344 | 519 | 0.00026 | 0.00026 | 20562084.61 | 1036199519.00 | 50.39 | 0.9992 |
| 4 | 20561825 | 395 | 0.00019 | 0.00019 | 20561627.51 | 1015637434.00 | 49.39 | 0.9991 |
| 5 | 20561430 | 346 | 0.00017 | 0.00017 | 20561257.15 | 995075806.90 | 48.40 | 0.9991 |
| 6 | 20561084 | 270 | 0.00013 | 0.00013 | 20560949.47 | 974514549.70 | 47.40 | 0.9991 |
| 7 | 20560815 | 261 | 0.00013 | 0.00013 | 20560684.17 | 953953600.30 | 46.40 | 0.9991 |
| 8 | 20560554 | 258 | 0.00013 | 0.00013 | 20560424.78 | 933392916.10 | 45.40 | 0.9991 |
| 9 | 20560296 | 242 | 0.00012 | 0.00012 | 20560174.77 | 912832491.30 | 44.40 | 0.9991 |
| 10 | 20560054 | 249 | 0.00012 | 0.00012 | 20559929.12 | 892272316.60 | 43.40 | 0.9990 |
| 11 | 20559805 | 258 | 0.00012 | 0.00012 | 20559675.60 | 871712387.40 | 42.40 | 0.9990 |
| 12 | 20559547 | 313 | 0.00015 | 0.00015 | 20559390.12 | 851152711.80 | 41.40 | 0.9990 |
| 13 | 20559234 | 393 | 0.00019 | 0.00019 | 20559036.89 | 830593321.70 | 40.40 | 0.9990 |
| 14 | 20558840 | 510 | 0.00024 | 0.00024 | 20558584.95 | 810034284.80 | 39.40 | 0.9990 |
| 15 | 20558330 | 797 | 0.00038 | 0.00038 | 20557931.28 | 789475699.90 | 38.40 | 0.9990 |
| 16 | 20557533 | 1022 | 0.00048 | 0.00048 | 20557021.77 | 768917768.60 | 37.40 | 0.9989 |
| 17 | 20556511 | 1351 | 0.00063 | 0.00063 | 20555835.16 | 748360746.80 | 36.41 | 0.9989 |
| 18 | 20555160 | 2328 | 0.00107 | 0.00107 | 20553995.76 | 727804911.70 | 35.41 | 0.9988 |
| 19 | 20552832 | 3031 | 0.00139 | 0.00138 | 20551316.26 | 707250915.90 | 34.41 | 0.9987 |
| 20 | 20549801 | 22633 | 0.00110 | 0.00110 | 20538483.91 | 686699599.70 | 33.42 | 0.9985 |
| 21 | 20527167 | 24454 | 0.00119 | 0.00119 | 20514940.30 | 666161115.70 | 32.45 | 0.9974 |
| 22 | 20502713 | 26534 | 0.00130 | 0.00129 | 20489446.49 | 645646175.40 | 31.49 | 0.9963 |
| 23 | 20476180 | 28872 | 0.00141 | 0.00141 | 20461743.81 | 625156729.00 | 30.53 | 0.9950 |
| 24 | 20447308 | 31546 | 0.00154 | 0.00154 | 20431534.91 | 604694985.10 | 29.57 | 0.9936 |
| 25 | 20415762 | 34575 | 0.00170 | 0.00169 | 20398474.07 | 584263450.20 | 28.62 | 0.9920 |
| 26 | 20381186 | 38016 | 0.00187 | 0.00187 | 20362178.28 | 563864976.20 | 27.67 | 0.9903 |
| 27 | 20343170 | 41965 | 0.00207 | 0.00206 | 20322187.54 | 543502797.90 | 26.72 | 0.9885 |
| 28 | 20301205 | 46457 | 0.00229 | 0.00229 | 20277976.48 | 523180610.30 | 25.77 | 0.9865 |
| 29 | 20254748 | 51624 | 0.00255 | 0.00255 | 20228935.96 | 502902633.90 | 24.83 | 0.9842 |
| 30 | 20203124 | 57537 | 0.00285 | 0.00285 | 20174355.24 | 482673697.90 | 23.89 | 0.9817 |
| 31 | 20145587 | 64343 | 0.00320 | 0.00319 | 20113415.25 | 462499342.70 | 22.96 | 0.9789 |
| 32 | 20081244 | 72183 | 0.00360 | 0.00360 | 20045152.61 | 442385927.40 | 22.03 | 0.9758 |
| 33 | 20009061 | 81232 | 0.00407 | 0.00406 | 19968445.59 | 422340774.80 | 21.11 | 0.9723 |
| 34 | 19927830 | 91696 | 0.00461 | 0.00460 | 19881982.06 | 402372329.20 | 20.19 | 0.9683 |
| 35 | 19836134 | 103808 | 0.00525 | 0.00523 | 19784230.48 | 382490347.20 | 19.28 | 0.9639 |
| 36 | 19732327 | 117843 | 0.00599 | 0.00597 | 19673404.99 | 362706116.70 | 18.38 | 0.9588 |
| 37 | 19614483 | 134153 | 0.00686 | 0.00684 | 19547406.66 | 343032711.70 | 17.49 | 0.9531 |
| 38 | 19480330 | 153095 | 0.00789 | 0.00786 | 19403782.47 | 323485305.00 | 16.61 | 0.9466 |
| 39 | 19327235 | 175118 | 0.00910 | 0.00906 | 19239675.79 | 304081522.60 | 15.73 | 0.9391 |
| 40 | 19152117 | 200727 | 0.01054 | 0.01048 | 19051752.92 | 284841846.80 | 14.87 | 0.9306 |
| Continued on next page | | | | |  | | | |

*Table 12 Continued from previous page*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | *lx* | *dx* | *mx* | *qx* | *Lx* | *Tx* | *ex* | *Sx* |
| 41 | 18951389 | 230476 | 0.01224 | 0.01216 | 18836151.08 | 265790093.90 | 14.02 | 0.9209 |
| 42 | 18720913 | 265029 | 0.01426 | 0.01416 | 18588398.50 | 246953942.80 | 13.19 | 0.9097 |
| 43 | 18455884 | 305073 | 0.01667 | 0.01653 | 18303347.48 | 228365544.30 | 12.37 | 0.8968 |
| 44 | 18150811 | 351331 | 0.01955 | 0.01936 | 17975145.41 | 210062196.80 | 11.57 | 0.8820 |
| 45 | 17799480 | 404559 | 0.02299 | 0.02273 | 17597200.21 | 192087051.40 | 10.79 | 0.8649 |
| 46 | 17394921 | 465428 | 0.02712 | 0.02676 | 17162206.66 | 174489851.20 | 10.03 | 0.8452 |
| 47 | 16929493 | 534496 | 0.03208 | 0.03157 | 16662245.00 | 157327644.50 | 9.29 | 0.8226 |
| 48 | 16394997 | 612031 | 0.03805 | 0.03733 | 16088981.44 | 140665399.50 | 8.58 | 0.7967 |
| 49 | 15782966 | 697855 | 0.04522 | 0.04422 | 15434038.41 | 124576418.10 | 7.89 | 0.7669 |
| 50 | 15085111 | 791034 | 0.05386 | 0.05244 | 14689593.93 | 109142379.70 | 7.24 | 0.7330 |
| 51 | 14294077 | 889647 | 0.06426 | 0.06224 | 13849253.30 | 94452785.73 | 6.61 | 0.6946 |
| 52 | 13404430 | 990400 | 0.07676 | 0.07389 | 12909229.64 | 80603532.42 | 6.01 | 0.6513 |
| 53 | 12414029 | 1088310 | 0.09175 | 0.08767 | 11869874.67 | 67694302.78 | 5.45 | 0.6032 |
| 54 | 11325720 | 1176486 | 0.10968 | 0.10388 | 10737476.98 | 55824428.11 | 4.93 | 0.5503 |
| 55 | 10149234 | 1246279 | 0.13102 | 0.12280 | 9526094.49 | 45086951.13 | 4.44 | 0.4932 |
| 56 | 8902955 | 1287884 | 0.15625 | 0.14466 | 8259012.70 | 35560856.64 | 3.99 | 0.4326 |
| 57 | 7615070 | 1291625 | 0.18587 | 0.16961 | 6969257.99 | 27301843.94 | 3.59 | 0.3700 |
| 58 | 6323446 | 1250020 | 0.22025 | 0.19768 | 5698435.54 | 20332585.94 | 3.22 | 0.3073 |
| 59 | 5073426 | 1160234 | 0.25966 | 0.22869 | 4493308.44 | 14634150.41 | 2.88 | 0.2465 |
| 60 | 3913191 | 1026242 | 0.30415 | 0.26225 | 3400070.17 | 10140841.97 | 2.59 | 0.1901 |
| 61 | 2886949 | 859581 | 0.35346 | 0.29775 | 2457158.34 | 6740771.80 | 2.33 | 0.1403 |
| 62 | 2027368 | 677824 | 0.40697 | 0.33434 | 1688455.49 | 4283613.46 | 2.11 | 0.0985 |
| 63 | 1349543 | 500731 | 0.46368 | 0.37104 | 1099177.56 | 2595157.98 | 1.92 | 0.0656 |
| 64 | 848812 | 345312 | 0.52225 | 0.40682 | 676155.86 | 1495980.41 | 1.76 | 0.0412 |
| 65 | 503500 | 221901 | 0.58110 | 0.44072 | 392549.37 | 819824.55 | 1.63 | 0.0245 |
| 66 | 281599 | 132900 | 0.63856 | 0.47195 | 215148.73 | 427275.18 | 1.52 | 0.0137 |
| 67 | 148699 | 74345 | 0.69309 | 0.49997 | 111526.27 | 212126.45 | 1.43 | 0.0072 |
| 68 | 74354 | 38999 | 0.74339 | 0.52450 | 54854.62 | 100600.18 | 1.35 | 0.0036 |
| 69 | 35355 | 19287 | 0.78857 | 0.54551 | 25712.02 | 45745.56 | 1.29 | 0.0017 |
| 70 | 16069 | 9049 | 0.82816 | 0.56315 | 11544.19 | 20033.54 | 1.25 | 0.0008 |
| 71 | 7020 | 4055 | 0.86207 | 0.57771 | 4991.98 | 8489.35 | 1.21 | 0.0003 |
| 72 | 2964 | 1748 | 0.89053 | 0.58956 | 2090.49 | 3497.37 | 1.18 | 0.0001 |
| 73 | 1217 | 729 | 0.91399 | 0.59908 | 852.23 | 1406.88 | 1.16 | 0.0001 |
| 74 | 488 | 296 | 0.93302 | 0.60664 | 339.83 | 554.65 | 1.14 | 0.0000 |
| 75 | 192 | 118 | 0.94827 | 0.61259 | 133.11 | 214.82 | 1.12 | 0.0000 |
| 76 | 74 | 46 | 0.96032 | 0.61723 | 51.39 | 81.71 | 1.10 | 0.0000 |
| 77 | 28 | 18 | 0.96977 | 0.62083 | 19.62 | 30.31 | 1.07 | 0.0000 |
| 78 | 10 | 6 | 0.9771 | 0.623599 | 7.424830053 | 10.69198881 | 0.99 | 0.000001 |
| 79 | 4 | 2 | 0.982746 | 0.625718 | 2.790411182 | 3.267158755 | 0.8 | 0 |
| 80 | 1 | 0.9 | 0.987065 | 0.627331 | 0.4767470869 | 0.4767475731 | 0.31 | 0 |

**4.4.1 Estimated against expected life mortality**

Table 13: Estimated life expectancy against expected life expectancy

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | **MODELS** | |  |
| **Gender** | **Cohort** | **RH** | **CBD** | **Expected life expectancy** |
| ***Female*** | 1981 | 74.84 | 63.86 | 77.8 |
| 2002 | 77.14 | 53.80 | 79.6 |
| ***Male*** | 1981 | 70.76 | 60.87 | 70.4 |
| 2002 | 73.58 | 53.35 | 74.4 |

This shows the comparison of the life expectancies at age zero for both models against the Expected life expectancy obtained from the Human Mortality Database.   
RH model performed better as its values for all cohorts are closer to the expected unlike CBD which underperformed in the 2002 cohorts.

### 

### 4.5 Objective 3: To compare the mortality trends based on the two models

#### 4.5.1 1981 Cohort

Figure 11: Probability of dying within the next t years against age for 1981 cohort

For the probability of dying within the next t years for the 1981 cohort, as age increases, the probability of dying also rises significantly. This observed trend is a widely recognized occurrence stemming from changes in the body's physiology as individuals age, coupled with an increased vulnerability to health problems over time. This trend is consistent for both genders and across both models and is expected due to the natural aging process and the accumulation of health risks over time. Under the CBD model, there is little to no distinction between the male and female mortality as they age suggesting a simplified representation of gender-specific mortality patterns. This could be attributed to the model's underlying assumptions that may not fully capture gender disparities in mortality risk .Under the RH model, The gap between male and female mortality probabilities widens after around age 60 indicating a more elaborate depiction of gender-specific mortality trends, with females having lower mortality risk as expected. However, this model predicts a steeper increase in mortality risk for both genders after age 60 compared to CBD. RH also predicts a significantly higher infant mortality as compared to the CBD model with male being higher than female for both models. This suggests that the RH model considers both biological differences and social factors that affect infant survival.

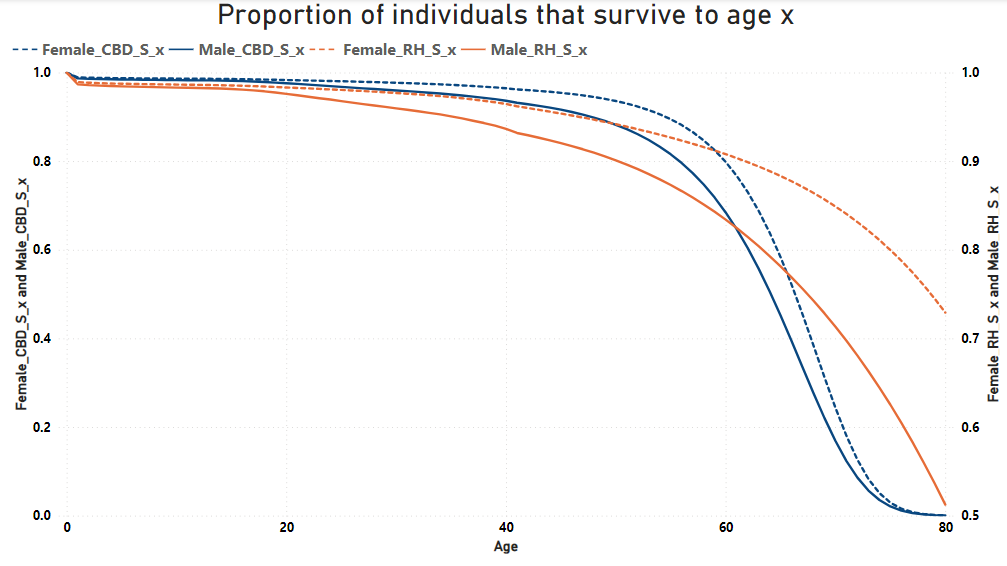


Figure 12:Proportion of individuals that survive to age x against age for 1981 cohort

Looking at the proportion of individuals that survive to age x in the 1981 cohort, the graph reveals that according to both models, females generally have higher survival proportions across all ages compared to males. Initially, there’s hardly any distinction between genders and models; however, disparities become more pronounced with increasing age. For both genders the CBD model predicts slightly higher survival proportions until around age 65. After 65 years of age, the RH model becomes more optimistic about survival proportions. For Female Survival Proportions both models start at a similar point but begin diverging around middle age.The CBD model predicts a steeper decline in survival proportion after middle age compared to the RH model. For Male Survival Proportions a similar pattern is observed as in females but at lower proportions throughout life. Males according to both models experience an accelerated decline in survival rates post-middle-age compared to females.

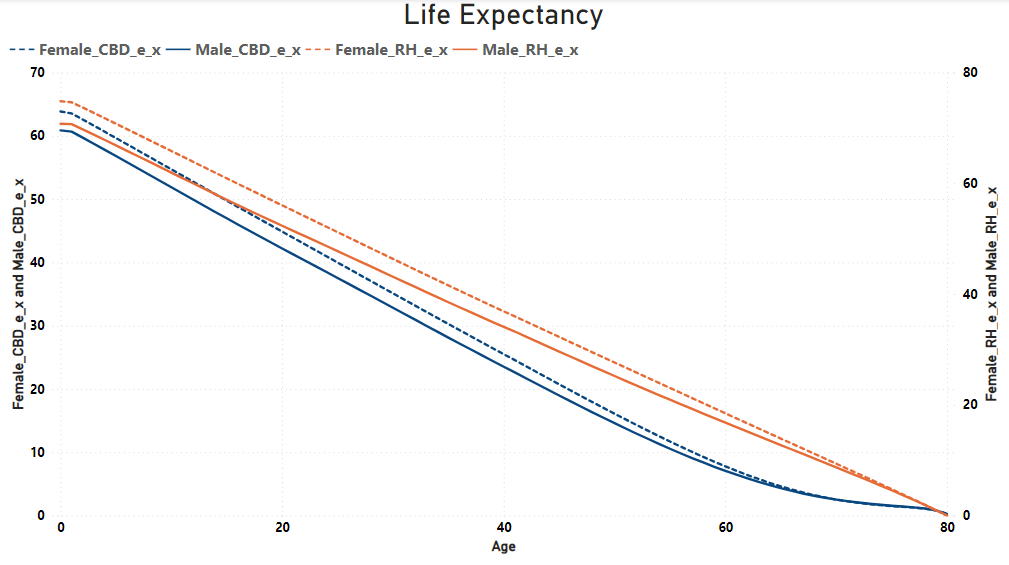


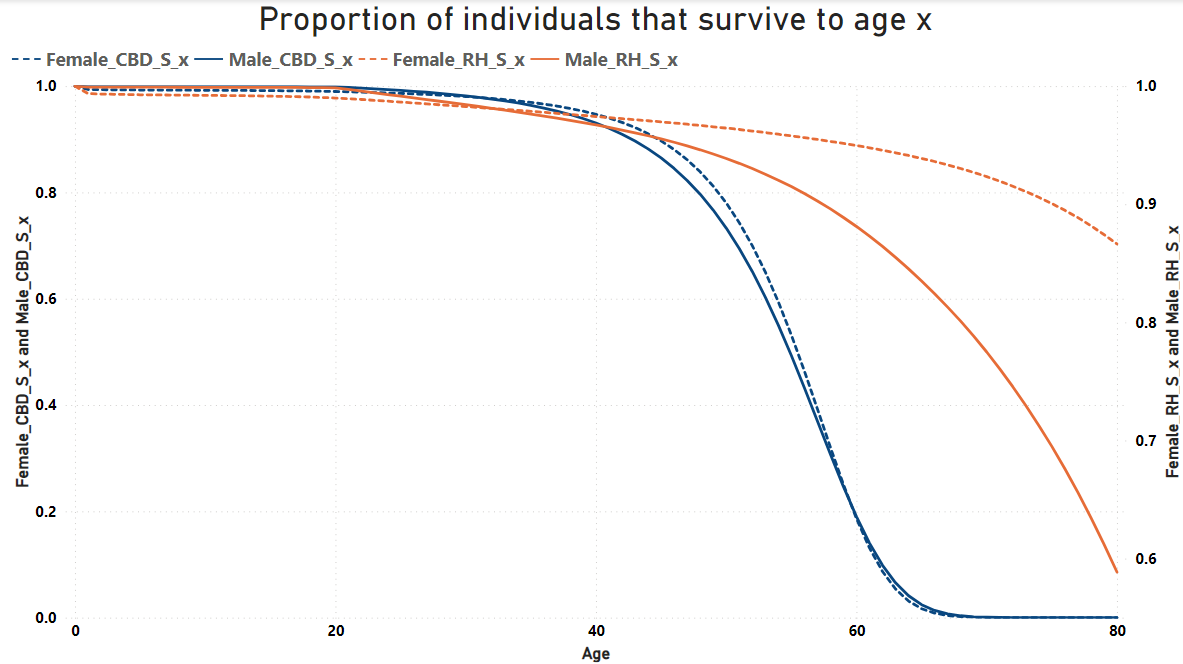
Figure 13: Life expectancy against age, for 1981 cohort

For life expectancy for the 1981 cohort, the RH model predicts higher life expectancies for both males and females compared to the CBD model for all ages.This discrepancy can be attributed to various factors, including differences in model assumptions, but more importantly, the RH model's higher life expectancies may reflect its incorporation of mortality improvements over time, capturing advancements in healthcare and changes in lifestyle behaviors.Under the both models, as expected, females consistently have slightly higher life expectancy than males across all ages. At around age 60 going forward, the male and female life expectancies start converging for both models.This could be attributed to the fact that socioeconomic factors and occupational risks may become less disparate among older individuals diminishing gender differences in mortality. The life expectancy as expected decreases with increase in age with CBD showing a more gradual change.

#### 4.5.2 2002 Cohort

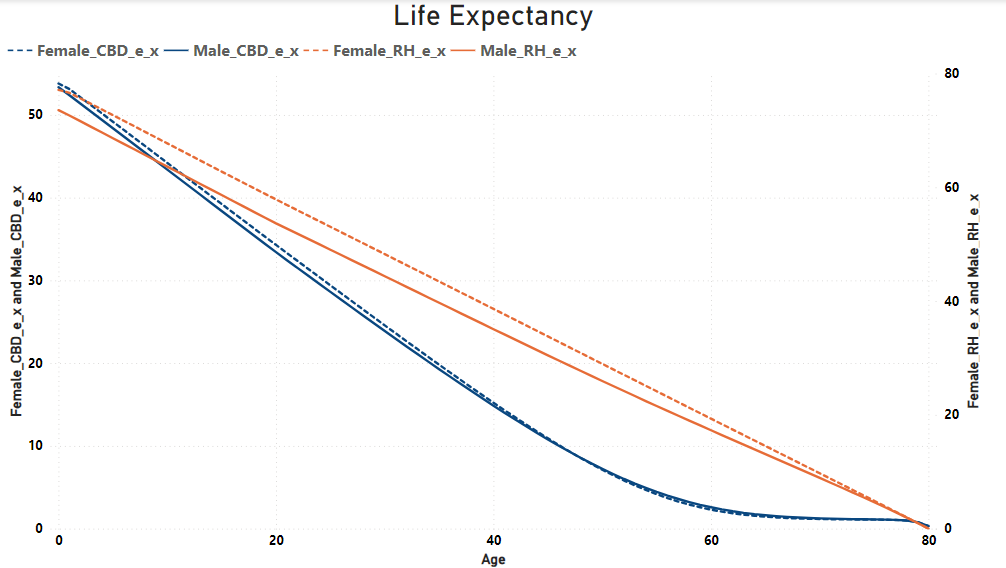
Figure 14: Probability of dying within the next t years, for 2002 cohort

In the 2002 cohort, the CBD model predicts a higher probability of dying compared to the RH model across all ages consistently raising the possibility that the CBD model may be overestimating mortality rates. .Under the CBD model,there is little to no distinction between the male and female mortality as they age suggesting a simplified representation of gender-specific mortality patterns.This could be attributed to the model's underlying assumptions that may not fully capture gender disparities in mortality risk .Both models show an increasing probability of dying as age progresses and this aligns with real-world mortality patterns, where risks rise with age probably due to health conditions and aging-related factors. CBD predicts a steeper increase in mortality risk with age as compared to RH. .Under the RH model, the gap between male and female mortality probabilities widens after around age 60 indicating a more elaborate depiction of gender-specific mortality trends,with females having lower mortality risk similar to the 1980 cohort. The RH model also predicts a higher probability of dying in infants of both genders. Both models also predict lower mortality rates for this cohort as compared to the 1981 cohort.

Figure 15: Proportion of individuals that survive to age x for 2002 cohort

The graph depicts the proportion of individuals who survived to various ages for the 2002 cohort. The CBD model predicts a higher survival probability across all ages with females consistently having a slightly higher survival rate than males under this model. The curve also shows a gradual decline in survival as age increases.The RH model is more conservative in its estimates. Initially, males and females have similar survival probabilities then at around age 60, females are forecasted to have a slightly better survival rate.The RH curve just like the CBD curve also declines with age.The decline in both models reflects increased mortality risk with age. RH shows a subtle gender difference after age 60.

Figure 16: Life expectancy against age for 2002 cohort



For the 2002 cohort, the CBD model initially predicts higher life expectancies for both males and females compared to the RH model but has a steeper drop resulting in higher RH life expectancies for most ages. The RH decline in life expectancy is more gradual throughout. Under the CBD model, females consistently have slightly higher life expectancy than males across all ages. The RH model shows almost identical life expectancies for males and females past age 70. However, females are forecasted to have a slightly higher life expectancy for most ages.

### 4.5.3 Longevity Risk

Figure 17: Life expectancy against time, for Longevity Risk

Longevity risk pertains to the possibility of individuals living beyond anticipated timelines, surpassing the financial provisions of both individuals and institutions responsible for disbursing retirement benefits, like pension funds or insurance firms. Longevity risk is closely associated with life expectancies since life expectancy is an estimation of the average number of years a person is expected to live. In this study, the longevity risk at age 60 across different years was considered. This is a crucial age with regards to retirement .While both models demonstrated an overall increasing trend in life expectancy, the CBD model consistently registered lower life expectancies compared to the RH model, especially in the aftermath of the COVID-19 pandemic around 2020. This discrepancy suggests that the CBD model may potentially be enhancing longevity risk by underestimating life expectancies, particularly during periods of significant mortality fluctuations. For the period around 2020, due to the COVID-19 pandemic, the mortality rates were highly impacted and hence had a great influence on the life expectancy leading to a sharp downward shift for both models with CBD model dipping further downward as compared to RH model.

After the 2020 period there was an increasing trend for both models. However, the CBD model registered lower life expectancies for the periods in comparison to the RH model. Both models portrayed improved life expectancies for the retirement ages for the recent years. The increasing trend was influenced by improved medical treatments, technologies, and healthcare practices that have led to better disease management, reduced mortality rates, and improved overall health outcomes. When life expectancy increased, it indicated that people are living longer lives, which can intensify the longevity risk, thus, the CBD model's tendency to portray lower life expectancies compared to the RH model highlights its potential role in enhancing longevity risk.

# 

# CHAPTER 5

## Summary, Conclusions and Recommendations

## 5.1 Summary

The main objective was to model mortality with the Cairns Blake Dowd and the Renshaw Haberman models and perform a comparative analysis of the mortality trends. To achieve this, the project was guided by three objectives which included the estimation of mortality rates, comparison of the mortality trends and construction of cohort life tables, using the mortality rates from both models.

The Renshaw Haberman (RH) model demonstrated superior predictive capability over the CBD model, as indicated by lower Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) values for both male and female cohorts in 1981 and 2002. Moreover, the RH model proved reliable even when provided with less data, outperforming the Cairns Blake Dowd (CBD) model in predicting mortality rates for the 2002 cohort. Notably, the RH model consistently exhibited accuracy in mortality rate predictions for both cohorts, while the CBD model faltered, particularly evident in the 2002 cohort where its accuracy notably declined with longer forecast periods.

Under mortality trends, both the CBD and RH models suggest that females generally have lower mortality risks and higher survival proportions compared to males across the lifespan of the 1981 cohort. The RH model predicts a steeper increase in mortality risk after age 60 and higher infant mortality rates compared to the CBD model. The RH model also forecasts higher life expectancies for both genders compared to the CBD model, with females consistently having slightly higher life expectancies under the CBD model but almost identical life expectancies with males under the RH model after around age 60.The CBD model for the 2002 cohort predicts a higher probability of dying across all ages compared to the RH model. Females generally face a slightly higher mortality risk than males under the CBD model, while the RH model assumes similar mortality risks for males and females until around age 60. Survival probabilities decline with increasing age in both models, with the RH model showing a more conservative estimate initially but similar trends in survival probabilities. Life expectancies are higher under the RH model for most ages, although females consistently have slightly higher life expectancies under the CBD model across all ages.

Between 2000 and around 2018, life expectancy increased for both the Renshaw Haberman (RH) and Cairns Blake Dowd (CBD) models. However, the COVID-19 pandemic in 2020 significantly impacted mortality rates, leading to a sharp decline in life expectancy for both models, with the CBD model experiencing a more pronounced decrease compared to the RH model. Post-2020, both models showed an upward trend in life expectancy, but the CBD model consistently reported lower life expectancies than the RH model. Despite this, both models demonstrated improved life expectancies for retirement ages in recent years. The increase in life expectancy suggested people are living longer, heightening longevity risk for retirement funds.

## 5.2 Conclusion

In conclusion, the study compared mortality modeling using the Cairns Blake Dowd (CBD) and Renshaw Haberman (RH) models. From the analysis the RH model exhibited superior predictive capability, with lower errors and more reliable performance, particularly for the 2002 cohort. Both models suggested lower mortality risks for females and predicted higher life expectancies with age. However, the RH model showed more accurate predictions, especially for longer forecast periods, and highlighted gender differences in mortality trends.

The better performance of the RH model can be attributed to the age specific sensitivity to time component, as the model consistently exhibits accuracy in mortality rate predictions for both cohorts, while the CBD model's accuracy notably declines with longer forecast periods due to lack of the same, particularly evident in the 2002 cohort.

## 5.3 Recommendations

The data used in this study was USA data from the Human Mortality Database, for future projects it would be better to use data from developing countries such as Kenya to help better the mortality studies and provide a better statistic that is more relevant to the Kenyan population. Since different countries are exposed to unique risks and conditions.

Advancements in mortality statistical software would help simplify mortality prediction and analysis as we really struggled in synchronizing data formats with the currently available software such as the StMoMo.

We also recommend a similar study on the same topic using machine learning techniques to enhance traditional statistical approaches. This will help us can unlock new insights, improve prediction accuracy, and advance our understanding of population health dynamics. This research has the potential to contribute significantly to the field of public health and inform evidence-based decision-making for healthcare policy and intervention strategies.

Conduct longitudinal studies to track mortality trends over time in developing countries. This longitudinal approach will enable the identification of emerging patterns and the assessment of the effectiveness of interventions aimed at improving population health and reducing mortality rates.

We also emphasize the importance of integrating emerging data sources, such as electronic health records, demographic surveillance systems, and mobile health technologies, into mortality modeling efforts as leveraging these data sources can provide real-time insights into population health dynamics, facilitate timely intervention strategies, and enhance the accuracy of mortality predictions.

## 5.4 Limitations

Although mortality data in the USA is generally more comprehensive and reliable compared to many other countries, there may still be issues with data quality such as inconsistencies and missing values for certain periods. Estimating the parameters of the models accurately was also challenging, particularly when dealing with limited data or when the mortality trends exhibit unexpected patterns.

CBD and Renshaw-Haberman models rely on certain assumptions about the underlying mortality processes and these assumptions may not always hold, leading to potential inaccuracies in the model predictions. The accuracy and reliability of our long-term projections require careful consideration of factors such as population growth, changing disease patterns, and evolving healthcare practices.

Our initial plan was to study the Kenyan population but the unavailability of data and data privacy issues hindered our attempt.

# REFERENCES

Ahmad, F. B. (2023). Provisional Mortality Data — United States, 2022. *MMWR. Morbidity and Mortality Weekly Report*, *72*(18). <https://doi.org/10.15585/mmwr.mm7218a3>

Brass, William. 1971. Mortality models and their uses in demography. *Transactions of the Faculty of Actuaries* 33: 123–142.

Brouhns, Natacha, Michel Denuit, and Jeroen K. Vermunt. 2002. Measuring the Longevity Risk in Mortality Projections. *Bulletin of the Swiss Association of Actuaries* 2: 105–30.

Cairns, A. J. G., Blake, D., Dowd, K., Coughlan, G. D., Epstein, D., Ong, A., & Balevich, I. (2009). A Quantitative Comparison of Stochastic Mortality Models Using Data From England and Wales and the United States. *North American Actuarial Journal*, *13*(1), 1–35. <https://doi.org/10.1080/10920277.2009.10597538>

Cunningham, R. M., Walton, M. A., & Carter, P. M. (2018). The Major Causes of Death in Children and Adolescents in the United States. *New England Journal of Medicine*, *379*(25), 2468–2475. <https://doi.org/10.1056/nejmsr1804754>

Cycles, T. text provides general information S. assumes no liability for the information given being complete or correct D. to varying update, & Text, S. C. D. M. up-to-Date D. T. R. in the. (2022). *Topic: Death in the U.S.* Statista. <https://www.statista.com/topics/1294/death/#topicOverview>

Deaton, Angus S., and Christina Paxson. 2004. Mortality, Income, and Income Inequality over Time in Britain and the United States. In *Perspectives on the Economics of Aging*. Chicago: University of Chicago Press

De Moivre, A. (1731). *Annuities upon lives, or, the valuation of annuities upon any number of lives, as also, of reversions: To which is added, an appendix concerning the expectations of life, and probabilities of survivorship*. London printed, and Dublin re-printed, by and for S. Fuller.

Giacometti, Rosella, Sergio Ortobelli, and Maria Ida Bertocchi. 2009. Impact of Different Distributional Assumptions in Forecasting Italian Mortality Rates. *Investment Management and Financial Innovations* 6: 65–72.

Glenn, N. D. (1977). *Cohort analysis*. Sage Publications.

Gompertz, B. (1825). XXIV. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. In a letter to Francis Baily, Esq. FRS &c. *Philosophical Transactions of the city of London*, (115), 513-583.

Hill, L., & Artiga, S. (2023, May 23). *What is Driving Widening Racial Disparities in Life Expectancy?* KFF. <https://www.kff.org/racial-equity-and-health-policy/issue-brief/what-is-driving-widening-racial-disparities-in-life-expectancy/>

Hummer, R. A., & Hernandez, E. M. (2013). The Effect of Educational Attainment on Adult Mortality in the United States. *Population Bulletin*, *68*(1), 1–16. <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4435622/>

Kibiwott Bett, N. (2017). Modeling and Forecasting Mortality and Longevity Risk Based on Insucient Data: Kenyan Population. *Research Report in Mathematics*, *18*. <http://erepository.uonbi.ac.ke/bitstream/handle/11295/101300/Bett_Modelling%20and%20Forecasting%20Mortality%20and%20Longevity%20Risk%20Based%20on%20Insufficient%20Data-%20Kenyan%20Population.pdf>

Korhonen, V. (2024, February). *Death rate by age and sex in the U.S. 2019*. Statista. <https://www.statista.com/statistics/241572/death-rate-by-age-and-sex-in-the-us/#:~:text=In%20the%20United%20States%20in>

Lauren Medina, Shannon Sabo, & Jonathan Vespa. (2020). Retrieved from https://www.census.gov/content/dam/Census/library/publications/2020/demo/p25-1145.pdf

Lee, Ronald D., and Lawrence R. Carter. 1992. Modeling and Forecasting US Sex Differentials in Mortality. *International Journal of Forecasting* 8: 393–411.

Mitchell, Daniel, Patrick Brockett, Rafael Mendoza-Arriaga, and Kumar Muthuraman. 2013. Modeling and Forecasting Mortality Rates. *Insurance: Mathematics and Economics* 52: 275–85.

Suzanne Davies Withers. (2009). Longitudinal Methods (Cohort Analysis, Life Tables). *Elsevier EBooks*, 285–292. <https://doi.org/10.1016/b978-008044910-4.00469-7>

Renshaw, Arthur E., and Steven Haberman. 2006. A Cohort-Based Extension to the Renshaw-Haberman Model for Mortality Reduction Factors. *Insurance: Mathematics and Economics* 38: 556–70.

Thelle, D. S., & Laake, P. (2015). Epidemiology. *Research in Medical and Biological Sciences*, 275–320. <https://doi.org/10.1016/b978-0-12-799943-2.00009-4>

UNICEF(2015). *KEN - UNICEF DATA* <https://data.unicef.org/country/ken/>

WEISS, H. (2023, November 1). *America’s Infant Mortality Rate Is Increasing*.

<https://time.com/6330531/us-infant-mortality-rate-increase/>

Estimated life tables for. (1994, September). https://www.nber.org/system/files/working\_papers/h0059/h0059.pdf

Products - life tables . (2022). Retrieved from https://www.cdc.gov/nchs/products/life\_tables.htm

U.S. life expectancy 1950-2024. (n.d.). Retrieved from https://www.macrotrends.net/global-metrics/countries/USA/united-states/life-expectancy

Wharton PPI. (2016, June 27). *Penn Wharton Budget Model*. Penn Wharton Budget Model. <https://budgetmodel.wharton.upenn.edu/issues/2016/1/25/mortality-in-the-united-states-past-present-and-future>

Wilmoth, John R. 1990. Variation in vital rates by age, period, and cohort. *Sociological methodology* 20: 295–335.

Worldometers.*Kenya COVID - Coronavirus Statistics* [Kenya COVID - Coronavirus Statistics - Worldometer (worldometers.info)](https://www.worldometers.info/coronavirus/country/kenya/)

*Waruru, A., Onyango, D., Nyagah, L., Sila, A., Waruiru, W., Sava, S., Oele, E., Nyakeriga, E., Muuo, S. W., Kiboye, J., Musingila, P. K., van der Sande, M. A. B., Massawa, T., Rogena, E. A., DeCock, K. M., & Young, P. W. (2022). Leading causes of death and high mortality rates in an HIV-endemic setting (Kisumu County, Kenya, 2019). PLOS ONE, 17(1), e0261162.* [*https://doi.org/10.1371/journal.pone.0261162*](https://doi.org/10.1371/journal.pone.0261162)

Wunsch, G. (2012). *Introduction to Demographic Analysis*. Springer Science & Business Media.

Xu, J., Murphy, S., Kochanek, K., & Arias, E. (2022). *Mortality in the United States, 2021*. <https://www.cdc.gov/nchs/data/databriefs/db456.pdf>

# R Code Excerpt

This section shows an R Code excerpt illustrating R commands and inputs that were used.

library(demography)

library(StMoMo)

***#Data extraction (Female data)***

USA\_DATA<-hmd.mx(country="USA", username="graceshiks74@gmail.com", password="@Grace4073", label="USA")

USA\_DATA

head(USA\_DATA)

usa\_female<-StMoMoData(data=USA\_DATA, series ="female",type="central")

head(usa\_female)

***#Data extraction (Male data)***

usa\_male<-StMoMoData(data=USA\_DATA, series ="male",type="central")

head(usa\_male)

par(mfrow = c(1, 2))

plot(USA\_DATA,series="female",main="Female death rates")

plot(USA\_DATA,series="male",main="Male death rates")

par(mfrow = c(1, 2))

plot(USA\_DATA,series="female",plot.type="time",main="Female death rates")

plot(USA\_DATA,series="male",plot.type="time",main="Male death rates")

***#Fitting Renshaw Haberman model***

LCfit <- fit(lc(), data =usa\_female, ages.fit = 0:80)

head(LCfit)

plot(LCfit)

wxt <- genWeightMat(0:80, usa\_female$years, clip = 3)

RHfit <- fit(rh(), data = usa\_female, ages.fit = 0:80,

wxt = wxt, start.ax = LCfit$ax,

start.bx = LCfit$bx, start.kt = LCfit$kt)

plot(RHfit,parametricbx=FALSE)

wxt

plot(LCfit)

fitted.rh <- fitted(RHfit, type = "rates")

fitted.rh

ruxt <- RHfit$Dxt / RHfit$Ext

***#Comparing the fitted vs. observed rates at random ages***

plot(RHfit$years, ruxt["40", ], xlab = "year", ylab = "death rate",

main = "fitted vs. observed rates at age 40")

lines(RHfit$years, fitted.rh["40", ])

forecast.rh<-forecast(RHfit,h=61)

forecast.rh

plot(forecast.rh)

forecast.rh$rates

***#Fitting CBD model***

M6 <- m6()

M6fit <- fit(M6, data = central2initial(usa\_female), ages.fit =0:80)

M6fit

plot(M6fit, parametricbx = FALSE)

plot(M6fit)

fit.cbd<- fitted(M6fit, type = "rates")

fit.cbd

cuxt <- M6fit$Dxt / M6fit$Ext

***#Comparing the fitted vs. observed rates at random ages***

plot(M6fit$years, cuxt["40", ], xlab = "year", ylab = "death rate",

main = "fitted vs. observed rates at age 40 CBD (0-80)")

lines(M6fit$years, fit.cbd["40", ])

forecast.cbd<-forecast(M6fit,h=61,gc.order = c(0,0,1),method="ML")

forecast.cbd

forecast.cbd$rates

***#Checking RMSE***

mxf<-usa\_female$Dxt/usa\_female$Ext

mxf

***#Extracting the observed data***

cohort1981F<-extractCohort(mxf,cohort=1981)

cohort2002F<-extractCohort(mxf,cohort=2002)

***#Extracting the fitted data(RH and CBD)***

cohort1981fittedRh<-extractCohort(fitted.rh,cohort=1981)

cohort2002fittedRh<-extractCohort(fitted.rh,cohort=2002)

***#CBD***

cohort1981FCBD<-extractCohort(mxf,cohort=1981)

cohort2002FCBD<-extractCohort(mxf,cohort=2002)

cohort1981fittedCBD<-extractCohort(fit.cbd,cohort=1981)

cohort2002fittedCBD<-extractCohort(fit.cbd,cohort=2002)

***#RMSE CBD 1981***

RMSE\_RH1981 <- sqrt(mean((cohort1981F-cohort1981fittedRh)^2))

RMSE\_RH1981

***#RMSE CBD 2002***

RMSE\_RH2002<-sqrt(mean((cohort2002F-cohort2002fittedRh)^2))

RMSE\_RH2002

***#RMSE RH 1981***

RMSE\_CBD1981 <- sqrt(mean((cohort1981FCBD -cohort1981fittedCBD)^2))

RMSE\_CBD1981

***#RMSE RH 2002***

RMSE\_CBD2002<-sqrt(mean((cohort2002FCBD-cohort2002fittedCBD)^2))

RMSE\_CBD2002

***# Create a data frame to store the RMSE values***

rmse\_table <- data.frame(Model = c("CBD\_1981","RH\_1981","CBD\_2002", "RH\_2002"),

RMSE = c(RMSE\_CBD1981, RMSE\_RH1981,RMSE\_CBD2002, RMSE\_RH2002))

***# Print the table***

print(rmse\_table)

***#Checking MAE***

***#MAE CBD 1981***

MAE\_CBD1981 <- mean(abs(cohort1981FCBD -cohort1981fittedCBD))

MAE\_CBD1981

***# MAE CBD 2002***

MAE\_CBD2002 <- mean(abs(cohort2002FCBD-cohort2002fittedCBD))

MAE\_CBD2002

***# MAE RH 1981***

MAE\_RH1981 <- mean(abs(cohort1981F-cohort1981fittedRh))

MAE\_RH1981

***# MAE RH 2002***

MAE\_RH2002 <- mean(abs(cohort2002F-cohort2002fittedRh))

MAE\_RH2002

***# Create a data frame to store the MAE values***

Femalemae\_table <- data.frame(Model = c("CBD\_1981", "RH\_1981", "CBD\_2002", "RH\_2002"),

MAE = c(MAE\_CBD1981, MAE\_RH1981, MAE\_CBD2002, MAE\_RH2002))

***# Print the table***

print(Femalemae\_table)

***#Female Residuals***

RH\_Residuals<-residuals(RHfit)

RH\_Residuals

CBD\_Residuals<-residuals(M6fit)

CBD\_Residuals

plot(RH\_Residuals,type="scatter")

plot(CBD\_Residuals,type="scatter")

# 

# WORK PLAN

Figure 18: Work plan

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Activity** | **Sep 2023** | **Oct 2023** | **Nov 2023** | **Dec 2023** | **Jan 2024** | **Feb 2024** | **Mar 2024** |
| Study the theoretical Background |  |  |  |  |  |  |  |
| Literature Review |  |  |  |  |  |  |  |
| Methodology and proposal presentation |  |  |  |  |  |  |  |
| Data analysis and Findings |  |  |  |  |  |  |  |
| Project report |  |  |  |  |  |  |  |

# 

# 

# 

# BUDGET

Figure 19: Budget

|  |  |  |  |
| --- | --- | --- | --- |
| **ITEM** | **DESCRIPTION** | **QUANTITY** | **TOTAL COST(Ksh)** |
| Proposal | Printing copies of the research proposals | 6 | 1000 |
| Internet | Monthly subscription | 7 months | 14000 |
| Data collection | Data sourcing  Transport |  | 2000 |
| Research project | Printing copies of the final document | 6 | 1500 |
| **Total cost** |  |  | 18500 |